CHAPTER 24
TRANSMISSION LINES

GENERAL

The equations and charts of this chapter are for transmission lines operating in the TEM mode.* At the beginning of several of the sections (e.g., "Fundamental Quantities," "Voltage and Current," "Impedance and Admittance," "Reflection Coefficient") there are accurate equations, according to conventional transmission-line theory. These are applicable from the lowest power and communication frequencies, including direct current, up to the frequency where a higher mode begins to appear on the line.

Following the accurate equations are others that are specially adapted for use in radio-frequency problems. In cases of small attenuation, the terms x^2 and higher powers in the expansion of expαx, etc., are neglected. Thus, when $\alpha x = (\alpha/\beta)\theta = 0.1$ neper (or about 1 decibel), the error in the approximate equations is of the order of 1 percent.

Much of the information is useful also in connection with special lines, such as those with spiral (helical) inner conductors, which function in a quasi-TEM mode; likewise for microstrip.

It should be observed that Z_0 and Y_0 are complex quantities and the imaginary part cannot be neglected in the accurate equations, unless preliminary examination of the problems indicates the contrary. Even when attenuation is small, $Z_0 = 1/Y_0$ must often be taken at its complex value, especially when the standing-wave ratio is high. In the first few pages of equations, the symbol R_0 is used frequently. However, in later charts and special applications, the conventional symbol Z_0 is used where the context indicates that the quadrature component need not be considered for the moment.

Rule of Subscripts and Sign Conventions

The equations for voltage, impedance, etc., are generally for the quantities at the input terminals of the line in terms of those at the output terminals (Fig. 1). In case it is desired to find the quantities at the output in terms of those at the input, it is simply necessary to interchange the subscripts 1 and 2 in the equations and to place a minus sign before x or θ. The minus sign may then be cleared through the hyperbolic or circular functions; thus

$$\sinh(-\gamma x) = -\sinh\gamma x, \quad \text{etc.}$$

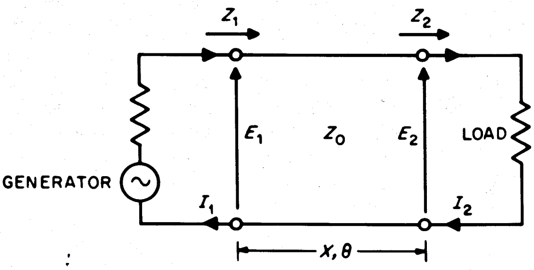

Fig. 1—Transmission line with generator, load.

SYMBOLS

Voltage and current symbols usually represent the alternating-current complex sinusoid, with magnitude equal to the root-mean-square value of the quantity.

Certain quantities, namely C, c, f, L, T, v, and ω are shown with an optional set of units in parentheses. Either the standard units or the optional units may be used, provided the same set is used throughout.

$A = 10\log_{10}(1/\eta)$ = dissipation loss in a length of line in decibels
$A_0 = 8.686\alpha x$ = normal or matched-line attenuation of a length of line in decibels.
B_0 = susceptive component of Y_0 in mhos
C = capacitance of line in farads/unit length (microfarads/unit length)
c = velocity of light in vacuum in units of length/second (units of length/microsecond). See page 3-11.
E = voltage (root-mean-square complex sinusoid) in volts
${}_fE$ = voltage of forward wave, traveling toward load
${}_rE$ = voltage of reflected wave

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* The information on pp. 24-1–24-18 is valid for single-mode waveguides in general, except for equations where the symbols R, L, G, or C per unit length are involved.

$|E_{\text{flat}}|$ = root-mean-square voltage when standing-wave ratio = 1.0
$|E_{\max}|$ = root-mean-square voltage at crest of standing wave
$|E_{\min}|$ = root-mean-square voltage at trough of standing wave
e = instantaneous voltage
$F_p = G/\omega C$ = power factor of dielectric
f = frequency in hertz (megahertz)
G = conductance of line in mhos/unit length
G_0 = conductive component of Y_0 in mhos
$g_a = Y_a/Y_0$ = normalized admittance at voltage standing-wave maximum
$g_b = Y_b/Y_0$ = normalized admittance at voltage standing-wave minimum
I = current (root-mean-square complex sinusoid) in amperes
${}_fI$ = current of forward wave, traveling toward load
${}_rI$ = current of reflected wave
i = instantaneous current
L = inductance of line in henries/unit length (microhenries/unit length)
P = power in watts
R = resistance of line in ohms/unit length
R_0 = resistive component of Z_0 in ohms
$r_a = Z_a/Z_0$ = normalized impedance at voltage standing-wave maximum
$r_b = Z_b/Z_0$ = normalized impedance at voltage standing-wave minimum
$S = |E_{\max}/E_{\min}|$ = voltage standing-wave ratio
T = delay of line in seconds/unit length (microseconds/unit length)
v = phase velocity of propagation in units of length/second (units of length/microsecond)
X_0 = reactive component of Z_0 in ohms
x = distance between points 1 and 2 in units of length (also used for normalized reactance $= X/Z_0$)
$Y_1 = G_1 + jB_1 = 1/Z_1$ = admittance in mhos looking toward load from point 1
$Y_0 = G_0 + jB_0 = 1/Z_0$ = characteristic admittance of line in mhos
$Z_1 = R_1 + jX_1$ = impedance in ohms looking toward load from point 1
$Z_0 = R_0 + jX_0$ = characteristic impedance of line in ohms
Z_{oc} = input impedance of a line open-circuited at the far end
Z_{sc} = input impedance of a line short-circuited at the far end
α = attenuation constant = nepers/unit length = 0.1151 × decibels/unit length
β = phase constant in radians/unit length
$\gamma = \alpha + j\beta$ = propagation constant
ϵ = base of natural logarithms = 2.718; or dielectric constant of medium (relative to air), according to context
$\eta = P_2/P_1$ = efficiency (fractional)
$\theta = \beta x$ = electrical length or angle of line in radians
$\theta^\circ = 57.3\theta$ = electrical angle of line in degrees
λ = wavelength in units of length
λ_0 = wavelength in free space
$\rho = |\rho| \angle 2\psi$ = voltage reflection coefficient
$\rho_{\text{dB}} = -20 \log_{10}(1/\rho)$ = voltage reflection coefficient in decibels
ϕ = time phase angle of complex voltage at voltage standing-wave maximum
ψ = half the angle of the reflection coefficient = electrical angle to nearest voltage standing-wave maximum on the generator side
$\omega = 2\pi f$ = angular velocity in radians/second (radians/microsecond).

FUNDAMENTAL QUANTITIES AND LINE PARAMETERS

$$dE/dx = -(R + j\omega L) I$$

$$d^2E/dx^2 = \gamma^2 E$$

$$dI/dx = -(G + j\omega C) E$$

$$d^2I/dx^2 = \gamma^2 I$$

$$\begin{aligned}\gamma = \alpha + j\beta &= [(R + j\omega L)(G + j\omega C)]^{1/2}\\ &= j\omega (LC)^{1/2}\\ &\quad \times [(1 - jR/\omega L)(1 - jG/\omega C)]^{1/2}\end{aligned}$$

$$\begin{aligned}\alpha = (\tfrac{1}{2}\{&[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}\\ &+ RG - \omega^2 LC\})^{1/2}\end{aligned}$$

$$\begin{aligned}\beta = (\tfrac{1}{2}\{&[(R^2 + \omega^2 L^2)(G^2 + \omega^2 C^2)]^{1/2}\\ &- RG + \omega^2 LC\})^{1/2}\end{aligned}$$

$$\begin{aligned}Z_0 = 1/Y_0 &= [(R + j\omega L)/(G + j\omega C)]^{1/2}\\ &= (L/C)^{1/2}[(1 - jR/\omega L)/(1 - jG/\omega C]^{1/2}\\ &= R_0(1 + jX_0/R_0)\end{aligned}$$

$$Y_0 = 1/Z_0 = G_0(1 + j\, B_0/G_0)$$

$$\alpha = \tfrac{1}{2}(R/R_0 + G/G_0)$$

$$\beta\, B_0/G_0 = \tfrac{1}{2}(R/R_0 - G/G_0)$$

$$R_0 = [M/2(G^2 + \omega^2 C^2)]^{1/2}$$

$$G_0 = [M/2(R^2 + \omega^2 L^2)]^{1/2}$$

$$B_0/G_0 = -X_0/R_0 = (\omega RC - \omega LG)/M$$

where

$$M=[(R^2+\omega^2L^2)(G^2+\omega^2C^2)]^{1/2}+RG+\omega^2LC$$

$$1/T=v=f\lambda=\omega/\beta$$

$$\beta=\omega/v=\omega T=2\pi/\lambda$$

$$\gamma x=\alpha x+j\beta x=(\alpha/\beta)\theta+j\theta$$

$$\theta=\beta x=2\pi\, x/\lambda=2\pi\, fTx$$

$$\theta^\circ=57.3\theta=360x/\lambda=360fTx.$$

(**A**) Special case—distortionless line: When $R/L=G/C$, the quantities Z_0 and α are independent of frequency.

$$X_0=0$$

$$\alpha=R/R_0$$

$$Z_0=R_0+j0=(L/C)^{1/2}$$

$$\beta=\omega(LC)^{1/2}.$$

(**B**) For small attenuation: $R/\omega L$ and $G/\omega C$ are small.

$$\gamma=j\omega(LC)^{1/2}\{1-j[(R/2\omega L)+(G/2\omega C)]\}$$

$$=j\beta(1-j\alpha/\beta)$$

$$\beta=\omega(LC)^{1/2}=\omega L/R_0=\omega CR_0$$

$$T=1/v=(LC)^{1/2}=R_0C$$

$$\alpha/\beta=(R/2\omega L)+(G/2\omega C)=(R/2\omega L)+\tfrac{1}{2}F_p$$

$$=(Rv/2\omega R_0)+\tfrac{1}{2}F_p$$

$$=\text{attenuation in nepers/radian}$$

$$=\frac{(\text{decibels per 100 feet})(\text{wavelength in line in meters})}{1663}$$

$$\alpha=\tfrac{1}{2}R(C/L)^{1/2}+\tfrac{1}{2}G(L/C)^{1/2}$$

$$=(R/2R_0)+\pi(F_p/\lambda)$$

$$=(R/2R_0)+\tfrac{1}{2}(F_p\beta)$$

where R and G vary with frequency, while L and C are nearly independent of frequency.

$$Z_0=1/Y_0$$

$$=(L/C)^{1/2}\{1-j[(R/2\omega L)-(G/2\omega C)]\}$$

$$=R_0(1+jX_0/R_0)$$

$$=1/[G_0(1+j\,B_0/G_0)]$$

$$=(1/G_0)(1-jB_0/G_0)$$

$$R_0=1/G_0=(L/C)^{1/2}$$

$$B_0/G_0=-(X_0/R_0)=(R/2\omega L)-\tfrac{1}{2}F_p=(\alpha/\beta)-F_p$$

$$X_0=-[R/2\omega(LC)^{1/2}]+(G/2\omega C)(L/C)^{1/2}$$

$$=-(R\lambda/4\pi)+(\tfrac{1}{2}F_p)R_0.$$

(**C**) With certain exceptions, the following few equations are for ordinary lines (e.g., not spiral delay lines) with the field totally immersed in a uniform dielectric of dielectric constant ϵ (relative to air). The exceptions are all the quantities not including the symbol ϵ, these being good also for special types such as spiral delay lines, microstrip, etc.

$$L=1.016R_0(\epsilon^{1/2})\times10^{-3}\text{ microhenries/foot}$$

$$=\tfrac{1}{3}R_0(\epsilon^{1/2})\times10^{-4}\text{ microhenries/centimeter}$$

$$C=1.016[(\epsilon^{1/2})/R_0]\times10^{-3}\text{ microfarads/foot}$$

$$=[(\epsilon^{1/2})/3R_0]\times10^{-4}\text{ microfarads/centimeter}$$

$$v/c=1016/R_0C'=\epsilon^{-1/2}$$

$=$velocity factor (with capacitance C' in picofarads/foot)

$$\lambda=\lambda_0v/c=c/f(\epsilon^{1/2})=\lambda_0/(\epsilon^{1/2})$$

$$T=1/v=R_0C'\times10^{-6}=1.016\times10^{-3}/(v/c)$$

$=1.016\times10^{-3}\epsilon^{1/2}$ microseconds/foot (with capacitance C' in picofarads/foot).

The line length is

$$x/\lambda=xf(\epsilon^{1/2})/984\text{ wavelengths}$$

$$\theta=2\pi x/\lambda=xf(\epsilon^{1/2})/156.5\text{ radians}$$

where xf is the product of feet times megahertz.

VOLTAGE AND CURRENT

$$E_1={}_fE_1+{}_rE_1={}_fE_2\epsilon^{\gamma x}+{}_rE_2\epsilon^{-\gamma x}$$

$$=E_2\{[(Z_2+Z_0)/2Z_2]\epsilon^{\gamma x}+[(Z_2-Z_0)/2Z_2]\epsilon^{-\gamma x}\}$$

$$=\tfrac{1}{2}(E_2+I_2Z_0)\epsilon^{\gamma x}+\tfrac{1}{2}(E_2-I_2Z_0)\epsilon^{-\gamma x}$$

$$=E_2[\cosh\gamma x+(Z_0/Z_2)\sinh\gamma x]$$

$$=E_2\cosh\gamma x+I_2Z_0\sinh\gamma x$$

$$=[E_2/(1+\rho_2)](\epsilon^{\gamma x}+\rho_2\epsilon^{-\gamma x})$$

$$I_1={}_fI_1+{}_rI_1={}_fI_2\epsilon^{\gamma x}+{}_rI_2\epsilon^{-\gamma x}$$

$$=Y_0({}_fE_2\epsilon^{\gamma x}-{}_rE_2\epsilon^{-\gamma x})$$

$$=I_2\{[(Z_0-Z_2)/2Z_0]\epsilon^{\gamma x}+[(Z_0+Z_2)/2Z_0]\epsilon^{-\gamma x}\}$$

$$=\tfrac{1}{2}(I_2+E_2Y_0)\epsilon^{\gamma x}+\tfrac{1}{2}(I_2-E_2Y_0)\epsilon^{-\gamma x}$$

$$=I_2[\cosh\gamma x+(Z_2/Z_0)\sinh\gamma x]$$

$$=I_2\cosh\gamma x+E_2Y_0\sinh\gamma x$$

$$=[I_2/(1-\rho_2)](\epsilon^{\gamma x}-\rho_2\epsilon^{-\gamma x})$$

$$E_1=AE_2+BI_2$$

$$I_1=CE_2+DI_2$$

where the general circuit parameters are $A=\cosh\gamma x$, $B=Z_0\sinh\gamma x$, $C=Y_0\sinh\gamma x$, and $D=\cosh\gamma x$.

Refer to section on "Matrix Algebra."

(A) When point 2 is at a voltage maximum or minimum, x' is measured from voltage maximum and x'' from voltage minimum (similarly for currents)

$$E_1=E_{\max}(\cosh\gamma x'+S^{-1}\sinh\gamma x')$$
$$=E_{\min}(\cosh\gamma x''+S\sinh\gamma x'')$$
$$I_1=I_{\max}(\cosh\gamma x'+S^{-1}\sinh\gamma x')$$
$$=I_{\min}(\cosh\gamma x''+S\sinh\gamma x'').$$

When attenuation is neglected

$$E_1=E_{\max}(\cos\theta'+j\,S^{-1}\sin\theta')$$
$$=E_{\min}(\cos\theta''+j\,S\sin\theta'').$$

(B) Letting $Z_l=$ impedance of load, $l=$ distance from load to point 2, and $x_l=$ distance from load to point 1

$$E_1=E_2\,\frac{\cosh\gamma x_l+(Z_0/Z_l)\sinh\gamma x_l}{\cosh\gamma l+(Z_0/Z_l)\sinh\gamma l}$$

$$I_1=I_2\,\frac{\cosh\gamma x_l+(Z_l/Z_0)\sinh\gamma x_l}{\cosh\gamma l+(Z_l/Z_0)\sinh\gamma l}.$$

(C) $e_1=\sqrt{2}\,|{}_fE_2|\,\epsilon^{\alpha x}\sin[\omega t+2\pi(x/\lambda)-\psi_2+\phi]$

$$+\sqrt{2}\,|{}_rE_2|\,\epsilon^{-\alpha x}\sin[\omega t-2\pi(x/\lambda)+\psi_2+\phi]$$

$i_1=\sqrt{2}\,|{}_fI_2|\,\epsilon^{\alpha x}$

$$\times\sin[\omega t+2\pi(x/\lambda)-\psi_2+\phi+\tan^{-1}(B_0/G_0)]$$
$$+\sqrt{2}\,|{}_rI_2|\,\epsilon^{-\alpha x}$$
$$\times\sin[\omega t-2\pi(x/\lambda)+\psi_2+\phi+\tan^{-1}(B_0/G_0)].$$

(D) For small attenuation

$$E_1=E_2\{[1+(Z_0/Z_2)\alpha x]\cos\theta$$
$$+j[(Z_0/Z_2)+\alpha x]\sin\theta\}$$
$$I_1=I_2\{[1+(Z_2/Z_0)\alpha x]\cos\theta$$
$$+j[(Z_2/Z_0)+\alpha x]\sin\theta\}.$$

(E) When attenuation is neglected

$$E_1=E_2\cos\theta+jI_2Z_0\sin\theta$$
$$=E_2[\cos\theta+j(Y_2/Y_0)\sin\theta]$$
$$={}_fE_2\epsilon^{j\theta}+{}_rE_2\epsilon^{-j\theta}$$
$$I_1=I_2\cos\theta+jE_2Y_0\sin\theta$$
$$=I_2[\cos\theta+j(Z_2/Z_0)\sin\theta]$$
$$=Y_0({}_fE_2\epsilon^{j\theta}-{}_rE_2\epsilon^{-j\theta}).$$

General circuit parameters are

$$A=\cos\theta$$
$$B=jZ_0\sin\theta$$
$$C=jY_0\sin\theta$$
$$D=\cos\theta.$$

IMPEDANCE AND ADMITTANCE

$$\frac{Z_1}{Z_0}=\frac{Z_2\cosh\gamma x+Z_0\sinh\gamma x}{Z_0\cosh\gamma x+Z_2\sinh\gamma x}$$

$$\frac{Y_1}{Y_0}=\frac{Y_2\cosh\gamma x+Y_0\sinh\gamma x}{Y_0\cosh\gamma x+Y_2\sinh\gamma x}.$$

(A) By interchange of subscripts and change of signs (see p. 24-1), the load impedance is

$$\frac{Z_2}{Z_0}=\frac{Z_1\cosh\gamma x-Z_0\sinh\gamma x}{Z_0\cosh\gamma x-Z_1\sinh\gamma x}.$$

For a length of uniform line or a symmetrical network

$$Z_1=Z_{oc}(Z_2+Z_{sc})/(Z_2+Z_{oc})$$
$$Z_2=Z_{oc}(Z_1-Z_{sc})/(Z_{oc}-Z_1).$$

(B) The input impedance of a line at a position of maximum or minimum voltage has the same phase angle as the characteristic impedance.

$$Z_1/Z_0=Z_b/Z_0=Y_0/Y_b=r_b+j0=S^{-1}$$

at a voltage minimum (current maximum).

$$Y_1/Y_0=Y_a/Y_0=Z_0/Z_a=g_a+j0=S^{-1}$$

at a voltage maximum (current minimum).

(C) When attenuation is small

$$\frac{Z_1}{Z_0}=\frac{[(Z_2/Z_0)+\alpha x]+j[1+(Z_2/Z_0)\alpha x]\tan\theta}{[1+(Z_2/Z_0)\alpha x]+j[(Z_2/Z_0)+\alpha x]\tan\theta}.$$

For admittances, replace Z_0, Z_1, and Z_2 by Y_0, Y_1, and Y_2, respectively.

When A and B are real

$$\frac{A\pm jB\tan\theta}{B\pm jA\tan\theta}=\frac{2AB\pm j(B^2-A^2)\sin2\theta}{(B^2+A^2)+(B^2-A^2)\cos2\theta}.$$

(D) When attenuation is neglected

$$\frac{Z_1}{Z_0}=\frac{Z_2/Z_0+j\tan\theta}{1+j(Z_2/Z_0)\tan\theta}=\frac{1-j(Z_2/Z_0)\cot\theta}{Z_2/Z_0-j\cot\theta}$$

and similarly for admittances.

(E) When attenuation $\alpha x=\theta\alpha/\beta$ is small and standing-wave ratio is large (say >10) (*Note*: The complex value of Z_0 or Y_0 must be used in computing the resistive component of Z_1 or Y_1.

For θ measured from a voltage minimum

$$Z_1/Z_0=[r_b+(\alpha/\beta)\theta](1+\tan^2\theta)+j\tan\theta$$

$$=[r_b+(\alpha/\beta)\theta](\cos^2\theta)^{-1}+j\tan\theta$$

(See Note 1)

$$\left.\begin{aligned} Z_0/Z_1&=Y_1/Y_0\\ &=[r_b+(\alpha/\beta)\theta](1+\cot^2\theta)-j\cot\theta\\ &=[r_b+(\alpha/\beta)\theta](\sin^2\theta)^{-1}-j\cot\theta.\end{aligned}\right\}$$

(See Note 2)

For θ measured from a voltage maximum

$$Z_0/Z_1=Y_1/Y_0=[g_a+(\alpha/\beta)\theta](1+\tan^2\theta)+j\tan\theta$$

(See Note 1)

$$Z_1/Z_0=[g_a+(\alpha/\beta)\theta](1+\cot^2\theta)-j\cot\theta.$$

(See Note 2)

Note 1: Not valid when $\theta\approx\pi/2$, $3\pi/2$, etc., due to approximation in denominator $1+(r_b+\theta\alpha/\beta)^2\tan^2\theta=1$ (or with g_a in place of r_b).

Note 2: Not valid when $\theta\approx0$, π, 2π, etc., due to approximation in denominator $1+(r_b+\theta\alpha/\beta)^2\cot^2\theta=1$ (or with g_a in place of r_b). For open- or short-circuited line, valid at $\theta=0$.

(**F**) When x is an integral multiple of $\lambda/2$ or $\lambda/4$: For $x=n\lambda/2$, or $\theta=n\pi$

$$\frac{Z_1}{Z_0}=\frac{(Z_2/Z_0)+\tanh n\pi(\alpha/\beta)}{1+(Z_2/Z_0)\tanh n\pi(\alpha/\beta)}.$$

For $x=n\lambda/2+\lambda/4$, or $\theta=(n+\frac{1}{2})\pi$

$$\frac{Z_1}{Z_0}=\frac{1+(Z_2/Z_0)\tanh(n+\frac{1}{2})\pi(\alpha/\beta)}{(Z_2/Z_0)+\tanh(n+\frac{1}{2})\pi(\alpha/\beta)}.$$

(**G**) For small attenuation, with any standing-wave ratio: For $x=n\lambda/2$, or $\theta=n\pi$, where n is an integer

$$\frac{Z_1}{Z_0}=\frac{(Z_2/Z_0)+n\pi(\alpha/\beta)}{1+(Z_2/Z_0)n\pi(\alpha/\beta)}$$

$$g_{a1}=\frac{g_{a2}+\alpha n\lambda/2}{1+g_{a2}\alpha n\lambda/2}=S_1^{-1}.$$

For $x=(n+\frac{1}{2})\lambda/2$, or $\theta=(n+\frac{1}{2})\pi$, where n is an integer or zero

$$\frac{Z_1}{Z_0}=\frac{1+(Z_2/Z_0)(n+\frac{1}{2})\alpha(\lambda/2)}{(Z_2/Z_0)+(n+\frac{1}{2})\alpha(\lambda/2)}$$

$$g_{b1}=\frac{1+g_{a2}(n+\frac{1}{2})(\alpha/\beta)\pi}{g_{a2}+(n+\frac{1}{2})(\alpha/\beta)\pi}=S_1.$$

Subscript a refers to the voltage-maximum point and b to the voltage minimum. In the above equations, subscripts a and b may be interchanged, and/or r may be substituted in place of g, except for the relationships to standing-wave ratio.

LINES OPEN- OR SHORT-CIRCUITED AT THE FAR END

Point 2 is the open- or short-circuited end of the line, from which x and θ are measured.

(**A**) Voltages and Currents: Use equations of "Voltages and Currents" section (p. 24-3), with the following conditions.

Open-circuited line:

$$\rho_2=1.00\angle 0°=1.00$$

$${}_rE_2={}_fE_2=E_2/2$$

$${}_rI_2=-{}_fI_2$$

$$I_2=0$$

$$Z_2=\infty.$$

Short-circuited line:

$$\rho_2=1.00\angle 180°=-1.00$$

$${}_rE_2=-{}_fE_2$$

$$E_2=0$$

$${}_rI_2={}_fI_2=I_2/2$$

$$Z_2=0.$$

(**B**) Impedances and admittances:

$$Z_{oc}=Z_0\coth\gamma x$$

$$Z_{sc}=Z_0\tanh\gamma x$$

$$Y_{oc}=Y_0\tanh\gamma x$$

$$Y_{sc}=Y_0\coth\gamma x.$$

(**C**) For small attenuation: Use equations for large swr in (**E**), p. 24-4, with the following conditions.

Open-circuited line:

$$g_a=0.$$

Short-circuited line:

$$r_b=0.$$

(**D**) When attenuation is neglected:

$$Z_{oc}=-jR_0\cot\theta$$

$$Z_{sc}=jR_0\tan\theta$$

$$Y_{oc}=jG_0\tan\theta$$

$$Y_{sc}=-jG_0\cot\theta.$$

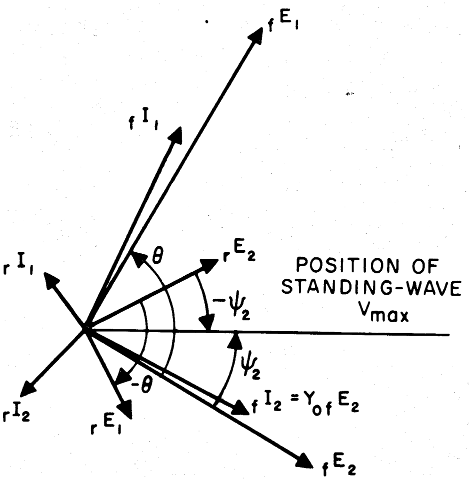

Fig. 2—Diagram of complex voltages and currents at two fixed points on a line with considerable attenuation. (Diagram rotates counterclockwise with time.)

(**E**) Relationships between Z_{oc} and Z_{sc}:

$$(Z_{oc}Z_{sc})^{1/2}=Z_0$$

$$\pm(Z_{sc}/Z_{oc})^{1/2}=\tanh\gamma x\approx(\alpha/\beta)\theta(1+\tan^2\theta)+j\tan\theta$$

$$=\frac{\alpha\theta}{\beta\cos^2\theta}+j\tan\theta$$

$$\approx j\tan\theta[1-j(\alpha/\beta)\theta(\tan\theta+\cot\theta)]$$

$$=j\tan\theta[1-j(\alpha/\beta)(2\theta/\sin 2\theta)].$$

Note: Above approximations not valid for $\theta\approx\pi/2$, $3\pi/2$, etc.

$$\pm(Z_{oc}/Z_{sc})^{1/2}=\coth\gamma x\approx(\alpha/\beta)\theta(1+\cot^2\theta)-j\cot\theta$$

$$=\frac{\alpha\theta}{\beta\sin^2\theta}-j\cot\theta$$

$$\approx -j\cot\theta[1+j(\alpha/\beta)\theta(\tan\theta+\cot\theta)]$$

$$=-j\cot\theta[1+j(\alpha/\beta)(2\theta/\sin 2\theta)].$$

Note: Above approximations not valid for $\theta\approx\pi$, 2π, etc.

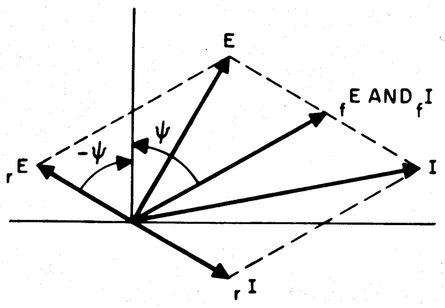

Fig. 3—Voltages and currents at time $t=0$ at a point ψ electrical degrees toward the load from a voltage standing-wave maximum.

(**F**) When attenuation is small (except for $\theta\approx n\pi/2$, $n=1, 2, 3, \cdots$)

$$\pm(Z_{sc}/Z_{oc})^{1/2}=\pm(Y_{oc}/Y_{sc})^{1/2}$$

$$=\pm j[-(C_{oc}/C_{sc})]^{1/2}$$

$$\times[1-j\tfrac{1}{2}(G_{oc}/\omega C_{oc}-G_{sc}/\omega C_{sc})]$$

where $Y_{oc}=G_{oc}+j\omega C_{oc}$ and $Y_{sc}=G_{sc}+j\omega C_{sc}$. The + sign is to be used before the radical when C_{oc} is positive, and the − sign when C_{oc} is negative.

(**G**) $R/|X|$ component of input impedance of low-attenuation nonresonant line:

Short-circuited line (except when $\theta\approx\pi/2$, $3\pi/2$, etc.)

$$R_1/|X_1|=G_1/|B_1|$$

$$=|(\alpha/\beta)\theta(\tan\theta+\cot\theta)+(B_0/G_0)|$$

$$=|(\alpha/\beta)(2\theta/\sin 2\theta)+(B_0/G_0)|.$$

Open-circuited line (except when $\theta\approx\pi$, 2π, etc.)

$$R_1/|X_1|=G_1/|B_1|$$

$$=|(\alpha/\beta)\theta(\tan\theta+\cot\theta)-(B_0/G_0)|$$

$$=|(\alpha/\beta)(2\theta/\sin 2\theta)-(B_0/G_0|.$$

VOLTAGE REFLECTION COEFFICIENT AND STANDING-WAVE RATIO

$$\rho={}_rE/{}_fE=-{}_rI/{}_fI=(Z-Z_0)/(Z+Z_0)$$

$$=(Y_0-Y)/(Y_0+Y)=|\rho|\angle 2\psi$$

where ψ is the electrical angle to the nearest voltage maximum on the generator side of point where ρ is measured (Figs. 2, 3, and 4).

$$\rho_1=\rho_2\epsilon^{-2\alpha x}\angle-2\theta$$

$$|\rho_1|=|\rho_2|/10^{A_0/10}.$$

Voltage reflection coefficient in decibels

$$\rho_{dB}=-20\log_{10}|1/\rho|.$$

The minus sign is frequently omitted.

$$|\rho_{dB}\text{ at input}|=|\rho_{dB}\text{ at load}|+2A_0.$$

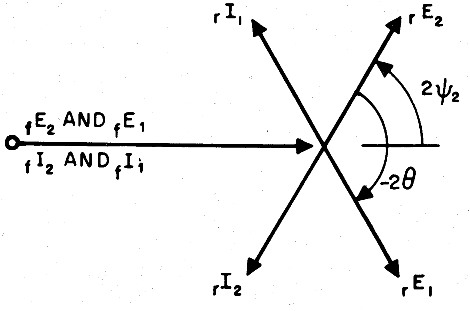

Fig. 4—Abbreviated diagram of a line with zero attenuation.

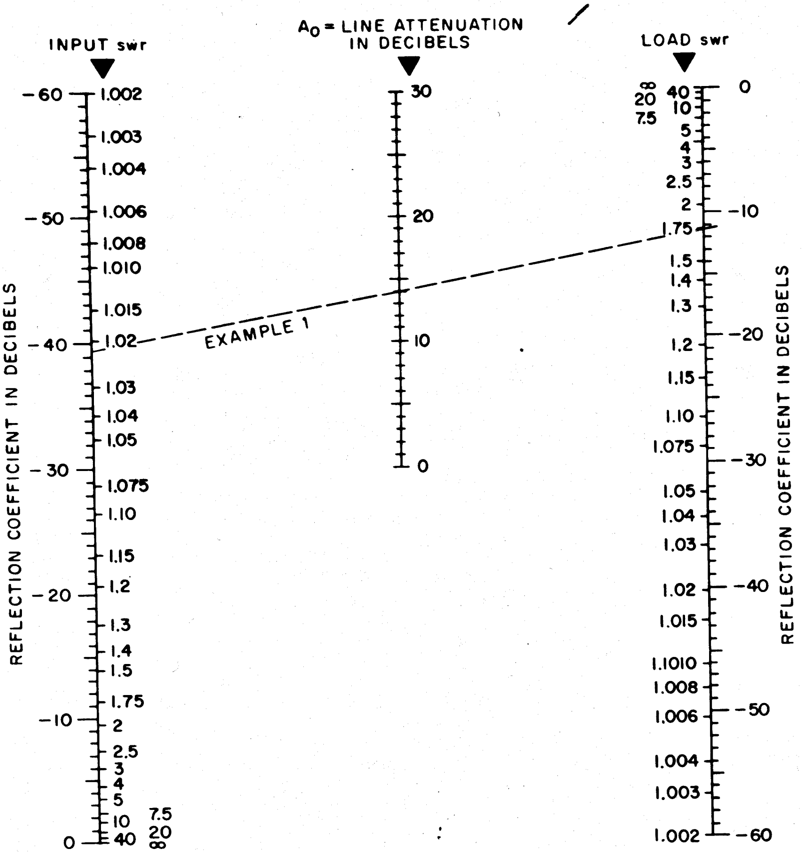

Fig. 5—Line attenuation and voltage reflection coefficient for low swr.

These two relationships and standing-wave ratio versus reflection coefficient in decibels are shown in Figs. 5 and 6.

$$Z = E/I = ({}_fE + {}_rE)/({}_fI + {}_rI)$$

$$= Z_0[(1+\rho)/(1-\rho)]$$

$$Z/Z_0 = (1+\rho)/(1-\rho)$$

$$= \frac{1+jS\cot\psi}{S+j\cot\psi}$$

$$S = |E_{\mathrm{max}}/E_{\mathrm{min}}| = |I_{\mathrm{max}}/I_{\mathrm{min}}|$$

$$= \frac{|{}_fE|+|{}_rE|}{|{}_fE|-|{}_rE|} = \frac{|{}_fI|+|{}_rI|}{|{}_fI|-|{}_rI|}$$

$$= \frac{1+|\rho|}{1-|\rho|} = r_a = g_a^{-1} = g_b = r_b^{-1}$$

$$|\rho| = (S-1)/(S+1)$$

$$1/S_1 = \tanh[\alpha x + \tanh^{-1}(1/S_2)]$$

$$= \tanh[0.1151A_0 + \tanh^{-1}(1/S_2)].$$

(**A**) For high standing-wave ratio. When the ratio S_1 is greater than 6/1, then with 1 percent accuracy or better

$$1/S_1 = 1/S_2 + \alpha x = 1/S_2 + 0.115A_0$$

$$|\rho_{\mathrm{dB}}| = 17.4/S.$$

Subject to the conditions below, the standing-wave ratio is given by one or the other of

$$S \approx (1+x^2)/r$$

$$S \approx (1+b^2)/g$$

where

$$r+jx = Z/Z_0 = (1/R_0)[R-(B_0/G_0)X+jX]$$

$$g+jb = Y/Y_0 = (1/G_0)[G+(B_0/G_0)B+jB].$$

Conditions, for 1-percent accuracy

$$r<0.1|x+1/x| \quad \text{when } |x|>0.3$$

$$g<0.1|b+1/b| \quad \text{when } |b|>0.3.$$

The boundary of the 1-percent-error region can be plotted on the Smith chart (Fig. 7) by use of the equation (for impedances)

$$|\cot\psi| = 0.1S^2/(S^2-1)^{1/2}.$$

The same boundary line on the chart holds when

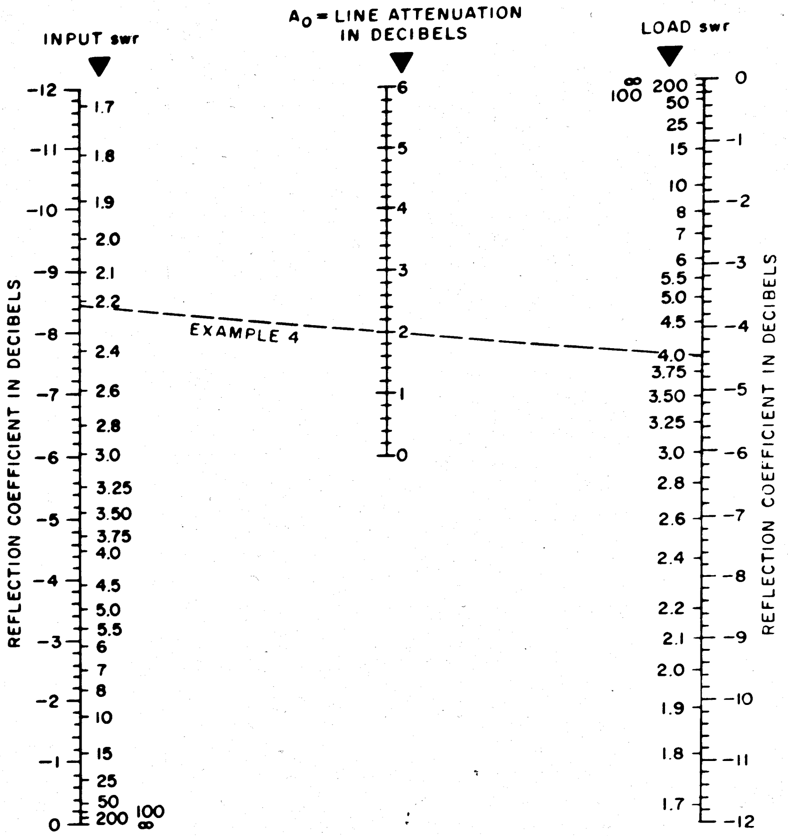

Fig. 6—Line attenuation and voltage reflection coefficient for high swr.

reading admittances. The area outside the solid heart-shaped curve is where the swr equation is accurate to within 1 percent. The area outside the dashed curve is where the reciprocal of $r+jx$ lies in the permitted region.

POWER AND EFFICIENCY

The net power flowing toward the load is

$$P=|{}_fE|^2G_0[1-|\rho|^2+2|\rho|(B_0/G_0)\sin 2\psi]$$

where $|E|$ is the root-mean-square voltage.

Example: Derive the power equation

$$P=(\text{real})\,EI^*.$$

When the following expressions are substituted in this equation, the power equation results.

$$E={}_fE(1+\rho)$$

$$I={}_fEY_0(1-\rho)$$

$$I^*={}_fE^*Y_0^*(1-\rho^*)$$

$$Y_0^*=G_0(1-jB_0/G_0)$$

$$\rho=|\rho|\exp j2\psi$$

$$\rho^*=|\rho|\exp -j2\psi.$$

(A) When the angle B_0/G_0 of the characteristic admittance is negligibly small, the net power flowing toward the load is given by

$$P=G_0(|{}_fE|^2-|{}_rE|^2)$$

$$=|{}_fE|^2G_0(1-|\rho|^2)$$

$$=|E_{\max}E_{\min}|/R_0$$

$$P_1=|{}_fE_2|^2G_0(\epsilon^{2(\alpha/\beta)\theta}-|\rho_2|^2\epsilon^{-2(\alpha/\beta)\theta}).$$

(B) Efficiency, when B_0/G_0 is negligibly small:

$$\eta=P_2/P_1=\frac{1-|\rho_2|^2}{\epsilon^{2(\alpha/\beta)\theta}-|\rho_2|^2\epsilon^{-2(\alpha/\beta)\theta}}$$

$$=\eta_{\max}\frac{1-|\rho_2|^2}{1-|\rho_2|^2\eta^2_{\max}}=\frac{1-|\rho_2|^2}{1-|\rho_1|^2}\epsilon^{-2\alpha x}$$

$$=\frac{1/|\rho_2|-|\rho_2|}{1/|\rho_1|-|\rho_1|}=\frac{S_1-1/S_1}{S_2-1/S_2}.$$

The maximum error in the above expressions is

$$\pm 100(S_2-1/S_2)B_0/G_0 \quad \text{percent}$$

$$\pm 4.34(S_2-1/S_2)B_0/G_0 \quad \text{decibels.}$$

When the ratio S_1 is greater than 6/1:

$$\eta\approx S_1/S_2\approx(1+0.115A_0S_2)^{-1}.$$

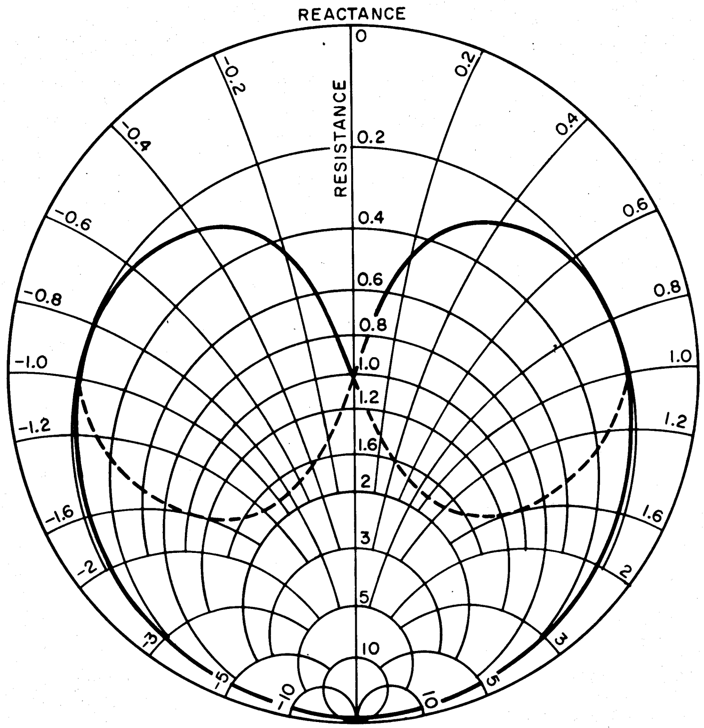

Fig. 7—Permitted region for use of equation $S\approx(1+x^2)/r$. *W. W. Macalpine, "Computation of Impedance and Efficiency of Transmission Line with High Standing-Wave Ratio," Trans. of the AIEE, vol. 72, part 1, p. 336, Fig. 2; July 1953.*

When the load matches the line, $\rho_2=0$ and the efficiency is accurately

$$\eta_{\max}=\exp[-2(\alpha/\beta)\theta]=\exp(-2\alpha x)=10^{-A_0/10}$$

$$A-A_0=10\log_{10}(\eta_{\max}/\eta).$$

Figure 8 is drawn from the expressions in this paragraph.

(**C**) Efficiency, when swr is high:

$$\eta=\frac{P_2}{P_1}=\frac{R_2}{R_1}\left(\frac{1+x_1^2}{1+x_2^2}\right)=\frac{G_2}{G_1}\left(\frac{1+b_1^2}{1+b_2^2}\right)$$

$$=\frac{R_2}{R_0^2G_1}\left(\frac{1+b_1^2}{1+x_2^2}\right)=\frac{R_0^2G_2}{R_1}\left(\frac{1+x_1^2}{1+b_2^2}\right)$$

where R is the ohmic resistance while x is the normalized reactance and similarly for G and b. It is important that the R's and G's be computed properly, using equations in the section on "Transformation of Impedance on Lines with High SWR," page 24-10. Note the identity of the efficiency equations with the left-hand terms of the impedance equations. The conditions for accuracy are the same as stated for the impedance equations for high standing-wave ratio.

Example: Physical significance of the equation for efficiency at high standing-wave ratio: Subject to stated conditions, approximately, $x=\cot\psi$ and $I=I_{\max}\sin\psi$. $I_{\max}$= current standing-wave maximum, practically constant along line when standing-wave ratio >6. Then

$$P=I^2R=I_{\max}{}^2R/(1+x^2).$$

When line length is greater than $\frac{1}{3}$ wavelength, then

$$\eta\approx[1+0.115A_0(1+x_2^2)(R_0/R_2)]^{-1}.$$

(**D**) Loss in nepers$=\frac{1}{2}\log_\epsilon(P_1/P_2)=0.1151\times$ (loss in decibels).

For a matched line, loss = attenuation $=(\alpha/\beta)\theta=\alpha x$ nepers.

Loss in decibels$=10\log_{10}(P_1/P_2)=8.686\times$ (loss in nepers).

When $2(\alpha/\beta)\theta$ is small

$$P_1/P_2=1+2(\alpha/\beta)\theta\,\frac{1+|\rho_2|^2}{1-|\rho_2|^2}$$

and

decibels/wavelength

$$=10\log_{10}\left(1+4\pi(\alpha/\beta)\,\frac{1+|\rho_2|^2}{1-|\rho_2|^2}\right).$$

(**E**) For the same power flowing in a line with standing waves as in a matched or "flat" line:

$$P=|E_{\text{flat}}|^2/R_0$$

$$|E_{\max}|=|E_{\text{flat}}|\,S^{1/2}$$

$$|E_{\min}|=|E_{\text{flat}}|/S^{1/2}$$

$$|{}_fE|=\tfrac{1}{2}\,|E_{\text{flat}}|\,(S^{1/2}+S^{-1/2})$$

$$|{}_rE|=\tfrac{1}{2}\,|E_{\text{flat}}|\,(S^{1/2}-S^{-1/2}).$$

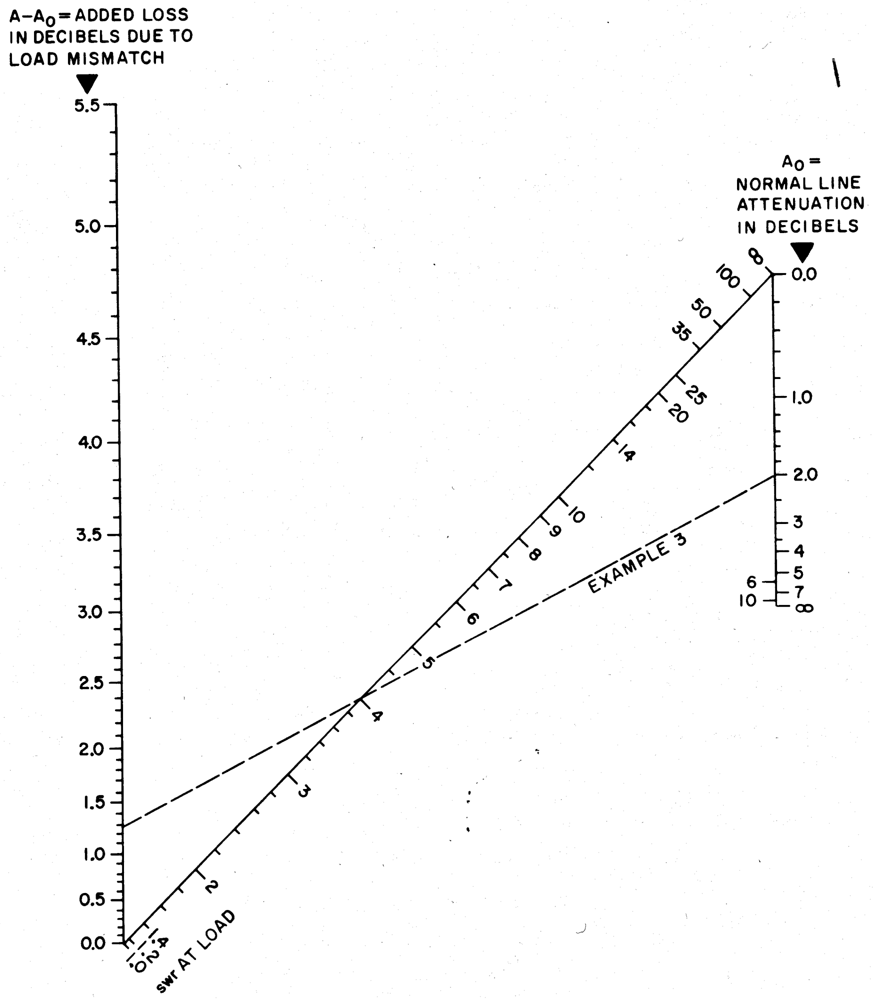

Fig. 8—Standing-wave loss factor. Due to load mismatch, an increase of loss in decibels as read from this figure must be added to normal line attenuation to give total dissipation loss in the line. This does not include mismatch loss due to any difference of line *input* impedance from the conjugate of the generator impedance [(***B***), p. 24-8].

When the loss is small, so that S is nearly constant over the entire length, then per half wavelength

$$(\text{power loss})/(\text{loss for flat line}) \approx \tfrac{1}{2}(S+1/S).$$

(**F**) The power dissipation per unit length, for unity standing-wave ratio, is

$$\Delta P_d/\Delta x = 2\alpha P$$

$$\frac{(\text{dissipation in watts/foot})}{(\text{line power in kilowatts})} = 2.30\ (\text{decibels/100 feet})$$

where the decibels/100 feet is the normal attenuation for a matched line.

When swr > 1, the dissipation at a current maximum is S times that for swr = 1, assuming the attenuation to be due to conductor loss only. The multiplying factor for local heating reaches a minimum value of $(S+1/S)/2$ all along the line when conductor loss and dielectric loss are equal.

(**G**) Further considerations on power and efficiency are given in "Mismatch and Transducer Loss," p. 24-11.

TRANSFORMATION OF IMPEDANCE ON LINES WITH HIGH SWR*

When standing-wave ratio is greater than 10 or 20, resistance cannot be read accurately on the Smith chart, although it is satisfactory for reactance.

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* W. W. Macalpine, "Computation of Impedance and Efficiency of Transmission Lines with High Standing-Wave Ratio," *Transactions of the AIEE*, vol. 72, part I, pp. 334–339; July 1953; also *Electrical Communication*, vol. 30, pp. 238–246; September 1953.

Use the equation (Fig. 9)

$$R_1 = R_2\frac{1+{x_1}^2}{1+{x_2}^2} + R_0(1+{x_1}^2)$$

$$\times\left[(\alpha/\beta)\theta + (B_0/G_0)\left(\frac{x_1}{1+{x_1}^2} - \frac{x_2}{1+{x_2}^2}\right)\right]$$

where R = ohmic resistance, $x = X/R_0$ = normalized reactance.

When admittance is given or required, similar equations can be written with the aid of the following tabulation. The top row shows the terms in the above equation.

R_1	R_2	${x_1}^2$	${x_2}^2$	R_0	x_1	$-x_2$
G_1	G_2	${b_1}^2$	${b_2}^2$	$1/R_0$	$-b_1$	b_2
R_1	$G_2{R_0}^2$	${x_1}^2$	${b_2}^2$	R_0	x_1	b_2
G_1	$R_2/{R_0}^2$	${b_1}^2$	${x_2}^2$	$1/R_0$	$-b_1$	$-x_2$

For transforming R to G or vice versa:

$$R = {R_0}^2 G\,|\,x/b\,|$$

where x and b are read on the Smith chart in the usual manner for transforming impedances to admittances.

The conditions for roughly 1-percent accuracy of the equations are:

Standing-wave ratio greater than 6/1 at input; $|\,B_0/G_0\,| < 0.1$; $r+jx$ or $g+jb$ (whichever is used, at each end of line) meet the requirements stipulated in paragraph (**A**) ("For high standing-wave ratio") on p. 24–7; and the line parameters and given impedance be known to 1-percent accuracy.

When line length is greater than $\frac{1}{3}$ wavelength, then

$$R_1 \approx R_2[(1+{x_1}^2)/(1+{x_2}^2)] + 0.115A_0R_0(1+{x_1}^2)$$

$$\frac{R_1/R_0}{1+{x_1}^2} \approx \frac{R_2/R_0}{1+{x_2}^2} + (\alpha/\beta)\theta.$$

The equation for resistance transformation is derived from expressions for high swr in paragraph (**A**), just referred to.

Example: A load of $0.4-j2000$ ohms is fed through a length of RG-218/U cable at a frequency of 2.0 megahertz. What are the input impedance and the efficiency for a 24-foot length of cable and for a 124-foot length?

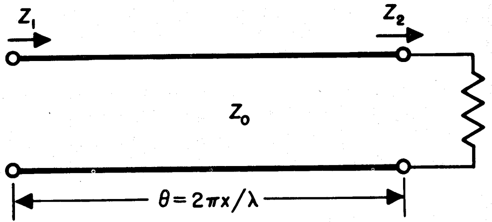


Fig. 9.

For RG-218/U, the attenuation at 2.0 megahertz is 0.095 decibel/100 feet (see Fig. 40). The dielectric constant $\epsilon = 2.26$ and F_p is negligibly small. Then, by equations in (**B**) and (**C**), p. 24–3:

$$B_0/G_0 = \alpha/\beta = (\text{dB}/100\text{ ft})\,(\lambda_{\text{meters}})/1663$$

$$= [0.095 \times 150/(2.26)^{1/2}]/1663 = 0.0057$$

$$x/\lambda = xf\epsilon^{1/2}/984 = 24 \times 2.0 \times 1.5/984 = 0.073$$

$$\theta = 2\pi x/\lambda = 0.46 \text{ radian for 24-foot length}$$

while

$$x/\lambda = 0.38 \text{ and } \theta = 2.4 \text{ for 124-foot length.}$$

$$Z_2/Z_0 \approx (0.4-j2000)/50 = 0.008-j40.$$

For the 24-foot length, by the Smith chart

$$x_1 = X_1/Z_0 = -1.9, \quad \text{or } X_1 = -95 \text{ ohms.}$$

The conditions for accuracy of the resistance transformation equation are satisfied. Now

$$1+{x_1}^2 = 1+(1.9)^2 = 4.6$$

$$1+{x_2}^2 = 1+(40)^2 = 1600$$

$$R_1 = 0.4(4.6/1600) + 50 \times 4.6 \times 0.0057$$

$$\times[0.46-(1.9/4.6)+(40/1600)]$$

$$= 0.0012 + 0.105 = 0.106 \text{ ohm.}$$

The efficiency equation in paragraph (**C**) on p. 24–9, gives

$$\eta = 0.0012/0.106 = 0.0113, \text{ or 1.1 percent}$$

where the 0.0012 figure is taken directly from the first quantity on the right-hand side of the computation of R_1.

Similarly, for the 124-foot length, $x_1 = 1.1$, $X_1 = 55$ ohms, $1+{x_1}^2 = 2.21$, $R_1 = 0.00055 + 1.83 = 1.83$ ohms

$$\eta = 0.00055/1.83 = 3.1 \times 10^{-4}, \text{ or 0.03 percent.}$$

Tabulating the results,

Length (feet)	Input Impedance (ohms)	Efficiency (%)	Loss (dB)
24	$0.106-j95$	1.1	19.6
124	$1.8\ +j55$	0.03	35

The considerably greater loss for 124 feet compared with 24 feet is because the transmission passes through a current maximum where the loss per unit length is much higher than at a current minimum.

MISMATCH AND TRANSDUCER LOSS

Figures 5, 6, and 8, plus the equations in this section, permit the calculation of loss when imped-

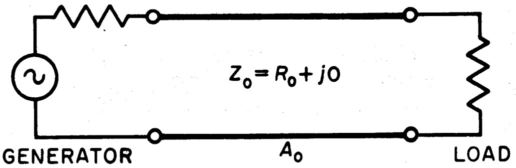


Fig. 10.

ance mismatch exists in a transmission-line system; also the change in standing-wave ratio along a line due to attenuation.

One End Mismatched

When either generator or load impedance is mismatched to the Z_0 of the line and the other is matched (Fig. 10)

$$(\text{mismatch loss}) = P_m/P$$

$$= 1/(1-|\rho|^2) = (S+1)^2/4S \quad (1)$$

where P= power delivered to load, P_m= power that would be delivered were system matched, and S= standing-wave ratio of mismatched impedance referred to Z_0.

Compared with an ideal transducer (ideal matching network between generator and load)

$$(\text{transducer loss}) = A_0 + 10\log_{10}(P_m/P) \text{ decibels} \quad (2)$$

where A_0= normal attenuation of line.

Generator and Load Mismatched

$$|X_0/R_0| \ll 1.$$

When mismatches exist at both ends of the system (Fig. 11)

$$(\text{mismatch loss at input})$$

$$= P_m/P$$

$$= [(R_g+R_1)^2 + (X_g+X_1)^2]/4R_gR_1 \quad (3)$$

$$(\text{transducer loss}) = (A-A_0) + A_0$$

$$+ 10\log_{10}(P_m/P) \text{ decibels} \quad (4)$$

where $(A-A_0)$ = standing-wave loss factor obtained from Fig. 8 for S= standing-wave ratio at load.

Notes on Equation (3)

This equation reduces to Eq. (1) when X_g and/or X_1 is zero.

In Eq. (3), the impedances can be either ohmic or normalized with respect to any convenient Z_0.

When determining input impedance R_1+jX_1 on Smith chart, adjust radius arm for S at input, determined from that at output by aid of Figs. 5 and 6.

For junction of two admittances, use Eq. (3) with G and B substituted for R and X, respectively.

Equation (3) is valid for a junction in any linear passive network. Likewise Eq. (1) when at least one of the impedances concerned is purely resistive. Determine S as if one impedance were that of a line.

Examples

Example 1: The swr at the load is 1.75 and the line has an attenuation of 14 decibels. What is the input swr?

Using Fig. 5, set a straightedge through the 1.75 division on the "load swr" scale and the 14-decibel point on the middle scale. Read the answer on the "input swr" scale, which the straightedge intersects at 1.022.

Example 2: Readings on a reflectometer show the reflected wave to be 4.4 decibels below the incident wave. What is the swr?

Using Fig. 6, locate the reflection coefficient 4.4 (or −4.4) decibels on either outside scale. Beside it, on the same horizontal line, read swr= 4.0+.

Example 3: A 50-ohm line is terminated with a load of $200+j0$ ohms. The normal attenuation of the line is 2.00 decibels. What is the loss in the line?

Using Fig. 8, align a straightedge through the points A_0= 2.0 and swr= 4.0. Read $A-A_0$= 1.27 decibels on the left-hand scale. Then the transmission loss in the line is

$$A = 1.27 + 2.00 = 3.27 \text{ decibels.}$$

This is the dissipation or heat loss as opposed to the mismatch loss at the input (example 4).

Example 4: In the preceding example, suppose the generator impedance is $100+j0$ ohms, and the line is 5.35 wavelengths long. What is the mismatch loss between the generator and the line?

According to example 3, the load swr= 4.0 and the line attenuation is 2.0 decibels. Then, using Fig. 6, the input swr is found to be 2.22. On the

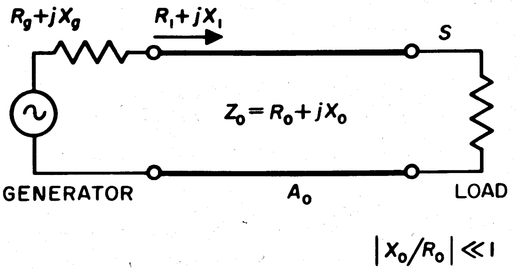


Fig. 11.

Smith chart, locate the point corresponding to 0.35 wavelength toward the generator from a voltage maximum, and swr = 2.22. Read the input normalized impedance as $0.62+j0.53$ with respect to $Z_0=50$ ohms. Now the mismatch loss at the input can be determined by use of (3). However, since the generator impedance is nonreactive, (1) can be used if desired. Refer to the following paragraph and to the "Notes on Equation (3)" above.

With respect to $100+j0$ ohms, the normalized impedance at the line input is $0.31+j0.265$, which gives swr = 3.5 according to the Smith chart. Then by (1), $P_m/P=1.45$, giving a mismatch loss of 1.62 decibels. The transducer loss is found by using the results of examples 3 and 4 in (4). This is

$$1.27+2.00+1.62=4.9 \text{ decibels.}$$

ATTENUATION AND RESISTANCE OF TRANSMISSION LINES AT ULTRA-HIGH FREQUENCIES

The normal or matched-line attenuation in decibels/100 feet is

$$A_{100}=4.34R_t/Z_0+2.78f\epsilon^{1/2}F_p$$

where the total line resistance/100 feet (for perfect surface conditions of the conductors) is, for copper coaxial line

$$R_t=0.1(1/d+1/D)f^{1/2}$$

and for copper 2-wire open line

$$R_t=(0.2/d)f^{1/2}$$

where D = diameter of inner surface of outer coaxial conductor in inches, d = diameter of conductors (coaxial-line center conductor) in inches, f = frequency in megahertz, ϵ = dielectric constant relative to air, and F_p = power factor of dielectric at frequency f.

For other conductor materials, the resistance of conductor of diameter d (and similarly for D) is

$$0.1(1/d)(f\mu_r\rho/\rho_{\text{Cu}})^{1/2} \text{ ohms/100 feet.}$$

Refer to section on "Skin Effect," in Chapter 6.

RESONANT LINES

Symbols

f_0 = resonance frequency in megahertz
G_a = conductance load in mhos at voltage standing-wave maximum, equivalent to some or all of the actual loads
k = coefficient of coupling
n = integral number of quarter wavelengths
$p=k^2Q_{1s}Q_{2s}$ = load transfer coefficient or matching factor
P_c = power converted into heat in resonator
P_m = power available from generator in watts $=E_{oc}^2/4R_{\text{gen}}$
P_x = power transferred when load is directly connected to generator (for single resonators); or an analogous hypothetical power (for two coupled resonators)
Q = figure of merit of a resonator as it exists, whether loaded or unloaded
Q_d = doubly loaded Q (all loads being included)
Q_s = singly loaded Q (all loads included except one). For a pair of coupled resonators, Q_{1s} is the value for the first resonator when isolated from the other. (Similarly for Q_{2s})
Q_u = unloaded Q
R_b = resistance load in ohms at voltage standing-wave minimum, equivalent to some or all of the actual loads
R_u = resistance similar to R_b except for unloaded resonator
R_1 = generator resistance, referred to short-circuited end
R_2 = load resistance
$S_x=R_1/R_2$ or R_2/R_1 = mismatch factor between generator and load
Z_{10} = characteristic impedance of the first of a pair of resonators
θ_1 = electrical angle from a voltage standing-wave minimum point

(**A**) Q of a resonator (electrical, mechanical or any other) is

$$Q=2\pi\frac{(\text{energy stored})}{(\text{energy dissipated per cycle})}$$

$$=2\pi f\frac{(\text{energy stored})}{(\text{power dissipation})}.$$

In a freely oscillating system, the amplitude decays exponentially.

$$I=I_0\exp(-\pi ft/Q).$$

(**B**) Unloaded Q of a resonant line:

$$Q_u=\beta/2\alpha$$

the line length being n quarter-wavelengths, where n is a small integer. The losses in the line are equivalent to those in a hypothetical resistor at the short-circuited end (**E**), p. 24-4:

$$R_u=n\pi Z_0/4Q_u.$$

(**C**) Loaded Q of a resonant line (Fig. 12):

$$Q^{-1}=Q_u^{-1}+(4R_b/n\pi Z_0)+(4G_a/n\pi Y_0)$$

$$=(4/n\pi Z_0)(R_u+R_b+G_a/Y_0^2).$$

All external loads can be referred to one end and represented by either R_b or G_a as in Fig. 13.

The total loading is the sum of all the individual loadings.

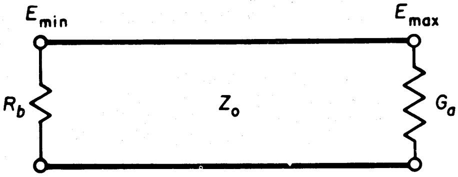


Fig. 12—Quarter-wave line with loadings at nominal short-circuit and open-circuit points.

General conditions:

$$R_b/Z_0 = G_a/Y_0 \ll 1.0$$

or, roughly, $Q > 5$.

(**D**) Input admittance and impedance:

The converse of the equations for Fig. 13 can be used at the resonance frequency. Then R or G is the input impedance or admittance, while

$$R_b = n\pi Z_0/4Q_s$$

where Q_s = singly loaded Q with the losses and all the loads considered except that at the terminals where input R or G is being measured.

In the vicinity of the resonance frequency, the input admittance when looking into a line at a tap point θ_1 in Fig. 14 is approximately

$$Y = G + jB = \frac{n\pi Y_0}{4\sin^2\theta_1}\left(Q_s^{-1} + j2\frac{f-f_0}{f_0}\right)$$

provided

$$|f-f_0|/f_0 \ll 1.0$$

and

$$|[\theta(f-f_0)/f_0]\cot\theta_1| \ll 1.0$$

where $\theta = n\pi/2$ = length of line at f_0. It is not valid when $\theta_1 \approx 0$, π, 2π, etc., except that it is good near the short-circuited end when $f-f_0 \approx 0$.

Such a resonant line is approximately equivalent to a lumped LCG parallel circuit, where

$$\omega_0^2 L_1 C_1 = (2\pi f_0)^2 L_1 C_1 = 1.$$

Admittance of the equivalent circuit is

$$Y = G + j[\omega C_1 - (1/\omega L_1)]$$

$$\approx \omega_0 C_1\{Q_s^{-1} + j2[(f-f_0)/f_0]\}.$$

Then, subject to the conditions stated above

$$L_1 = (4\sin^2\theta_1)/n\pi\omega_0 Y_0$$

$$C_1 = n\pi Y_0/(4\omega_0\sin^2\theta_1) = nY_0/(8f_0\sin^2\theta_1)$$

$$G = n\pi Y_0/(4Q_s\sin^2\theta_1)$$

$$Q_s = \omega_0 C_1/G = 1/\omega_0 L_1 G.$$

Similarly, the input impedance at a point in series with the line (Fig. 13C and D) is

$$Z = R + jX = \frac{n\pi Z_0}{4\cos^2\theta_1}\left(Q_s^{-1} + j2\frac{f-f_0}{f_0}\right)$$

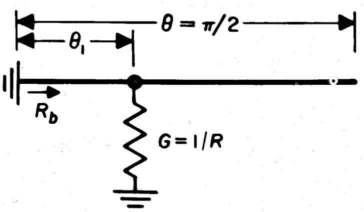
provided

$$|f-f_0|/f_0 \ll 1.0$$

and

$$|\theta[(f-f_0)/f_0]\tan\theta_1| \ll 1.0.$$

It is not valid when $\theta_1 \approx \pi/2$, $3\pi/2$, etc.

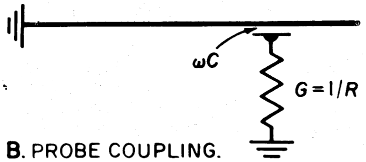


A. SHUNT OR TAPPED LOAD.

$R_b = (Z_0^2/R)\sin^2\theta_1$

OR

$G_a = G\sin^2\theta_1 = R_b/Z_0^2$



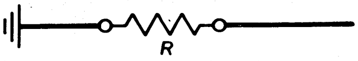
B. PROBE COUPLING.

$G_a = (\omega^2 C^2/G)\sin^2\theta_1$

OR

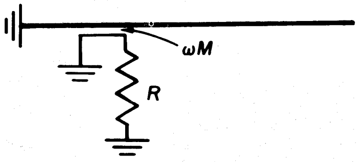
$R_b = Z_0^2\omega^2 C^2 R\sin^2\theta_1$

PROVIDED $G \gg \omega^2 C^2$



C. SERIES LOAD.

$R_b = R\cos^2\theta_1$



D. LOOP COUPLING.

$R_b = (\omega^2 M^2/R)\cos^2\theta_1$

PROVIDED $X_{loop} \ll R$

Fig. 13—Typical loaded quarter-wave sections with apparent R_b equivalent to the loading at distance θ_1 from voltage-minimum point of the line. Outer conductor not shown.

The voltage standing-wave ratio at resonance, on the generator (Fig. 15) is

$$S=(R_2+R_u)/R_1$$

$$=\frac{(R_2/R_1)Q_u+Q_d}{Q_u-Q_d}.$$

When $R_1=R_2$

$$S=\frac{1+Q_d/Q_u}{1-Q_d/Q_u}$$

$$\rho=Q_d/Q_u.$$

(E) Insertion loss (Fig. 15): At resonance, for either a distributed or a lumped-constant device

(dissipation loss)

$$=10\log_{10}(P_x/P_{\text{out}})$$

$$=20\log_{10}[1/(1-Q_d/Q_u)]$$

$$\approx 20\log_{10}(1+Q_d/Q_u)$$

$$\approx 8.7Q_d/Q_u \text{ decibels}$$

(mismatch loss)

$$=10\log_{10}(P_m/P_x)$$

$$=10\log_{10}[(1+S_x)^2/4S_x] \text{ decibels.}$$

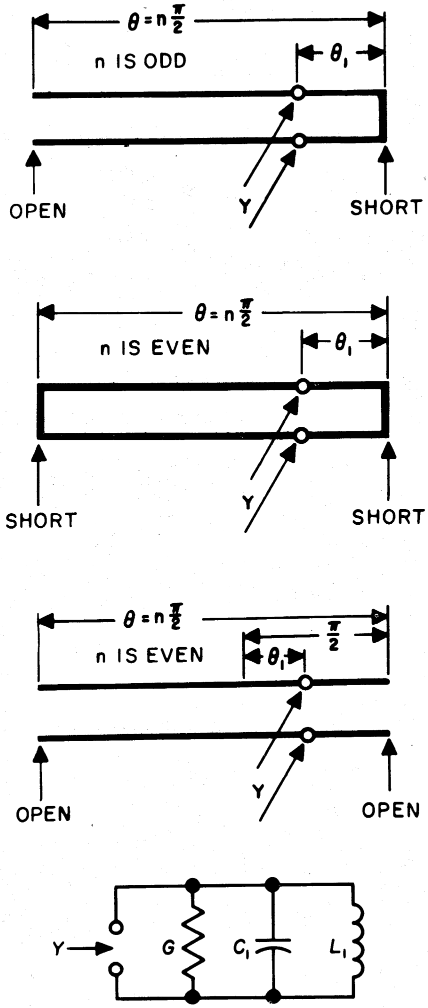


Fig. 14—Resonant transmission lines and their equivalent lumped circuit.

The dissipation loss also includes a small additional mismatch loss due to the presence of the resonator. The error in the form $20\log_{10}(1+Q_d/Q_u)$ is about twice that of the form $8.7Q_d/Q_u$. The last expression ($8.7Q_d/Q_u$) is in error compared with the first, $20\log_{10}[1/(1-Q_d/Q_u)]$, by roughly $-50(Q_d/Q_u)$ percent for $(Q_d/Q_u)<0.2$.

The selectivity is given on page 9-7, where $Q=Q_d$. That equation is accurate over a smaller range of $(f-f_0)$ for a resonant line than it is for a single tuned circuit.

At resonance*

$$P_{\text{out}}/P_{\text{in}}=R_2/(R_u+R_2)$$

$$=\frac{Q_u-Q_d}{Q_u+(R_1/R_2)Q_d}=1-Q_s/Q_u$$

where Q_s is for the resonator loaded with R_2 only.

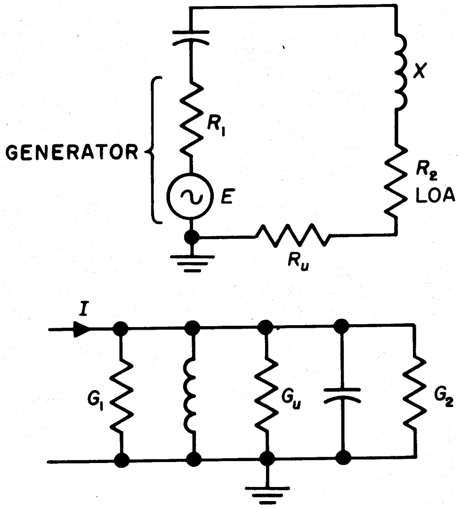


Fig. 15—Equivalent circuits of a resonant line (or a lumped tuned circuit) as seen at the short-circuited and open-circuited ends. All the power equations are good for either lumped or distributed parameters.

* When the line is resonated by a reactive load ($\theta \neq n\pi/2$), it is frequently preferable to use the resistance form of the equation. Compute R_u by the method given on p. 24-11, or on p. 24-5, where $Z_0=R_0(1-jB_0/G_0)$.

The maximum power transfer, for fixed Q_u, Q_d, and Z_0 occurs when $R_1 = R_2$. Then

$$P_{\text{out}}/P_{\text{in}} = (Q_u - Q_d)/(Q_u + Q_d) = 1 - Q_s/Q_u$$

$$P_{\text{out}}/P_m = (1 - Q_d/Q_u)^2$$

$$P_{\text{in}}/P_m = 1 - (Q_d/Q_u)^2.$$

When the generator R_1 or G_1 is negligibly small (then $Q = Q_s = Q_d$)

$$(P_{\text{in}}/P_{\text{out}})_s = Q_u/(Q_u - Q)$$

(F) Power dissipation $(= P_c)$:

$$P_c/P_m = \frac{4(Q_d/Q_u)(1 - Q_d/Q_u)}{1 + R_2/R_1}.$$

For matched input and output $(R_1 = R_2)$

$$P_c/P_m = 2(Q_d/Q_u)(1 - Q_d/Q_u)$$

$$\approx 2Q_d/Q_u \quad (\text{for } Q_d \ll Q_u)$$

$$P_c/P_{\text{out}} = 2Q_d/(Q_u - Q_d)$$

$$P_c/P_{\text{in}} = 2Q_d/(Q_u + Q_d).$$

For generator matched by load plus cavity

$$P_c/P_m = 2Q_d/Q_u.$$

When the generator R_1 or G_1 is negligibly small

$$(P_c/P_{\text{out}})_s = Q/(Q_u - Q)$$

$$(P_c/P_{\text{in}})_s = Q_s/Q_u.$$

(G) Voltage and current:

At the current-maximum point of an n-quarter-wavelength resonant line

$$I_{sc} = 4\left[\frac{P_m Q_d (1 - Q_d/Q_u)}{(1 + R_2/R_1) n\pi Z_0}\right]^{1/2}, \text{ rms amperes}$$

$$= 4\left[\frac{P_m Q_d}{\{1 + [(R_2 + R_u)/R_1]\} n\pi Z_0}\right]^{1/2}.$$

When the generator R_1 or G_1 is negligibly small

$$I_{sc} = 2\left[\frac{P_s Q_s}{n\pi Z_0 (R_2 + R_u)/R_s}\right]^{1/2}$$

where P_s = rated power of generator, R_s = rated load impedance as transformed into current-maximum point of cavity, and $I = I_{sc} \cos\theta_1$, while $E = Z_0 I_{sc} \sin\theta_1$.

The voltage and current are in quadrature time phase.

When $R_1 = R_2 + R_u$ and $n = 1$

$$I_{sc} \approx (8P_m Q_d/\pi Z_0)^{1/2}.$$

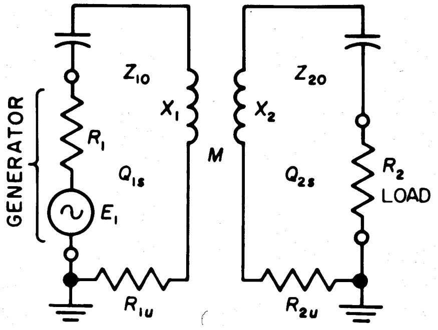
In a lumped-constant tuned circuit

$$I = 2\left[\frac{P_m Q_d (1 - Q_d/Q_u)}{(1 + R_2/R_1) X}\right]^{1/2}.$$

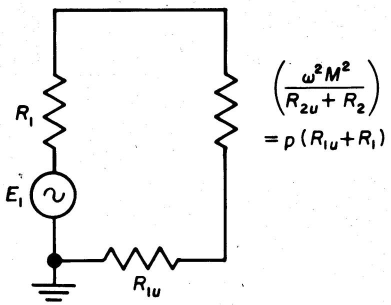
(H) Pair of coupled resonators (Fig. 16):

With inductive coupling near the short-circuited end of a pair of quarter-wave resonant lines

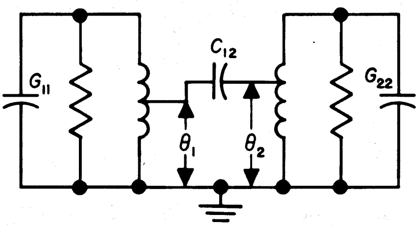
$$k = (4/\pi)\omega M/(Z_{10} Z_{20})^{1/2}.$$



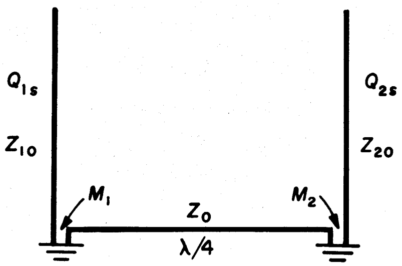
A. EQUIVALENT CIRCUIT WITH RESISTANCES AS SEEN AT THE SHORT-CIRCUITED END.



B. EQUIVALENT CIRCUIT OF FIRST RESONATOR AT RESONANCE FREQUENCY.



C. PROBE-COUPLED OR APERTURE-COUPLED RESONATORS.



D. QUARTER-WAVELENGTH LINE COUPLING.

Fig. 16—Two coupled resonators.

For coupling through a lossless quarter-wave-length line, inductively coupled near the short-circuited ends of the resonators (Fig. 16D):

$$k=\frac{4\omega^2 M_1 M_2}{\pi Z_0 (Z_{10} Z_{20})^{1/2}}.$$

Probe coupling near top (Fig. 16C):

$$k=(4/\pi)\omega C_{12}(Z_{10}Z_{20})^{1/2}\sin\theta_1\sin\theta_2.$$

For lumped-constant coupled circuits, p and k are defined on pp. 9-7 and 9-1.

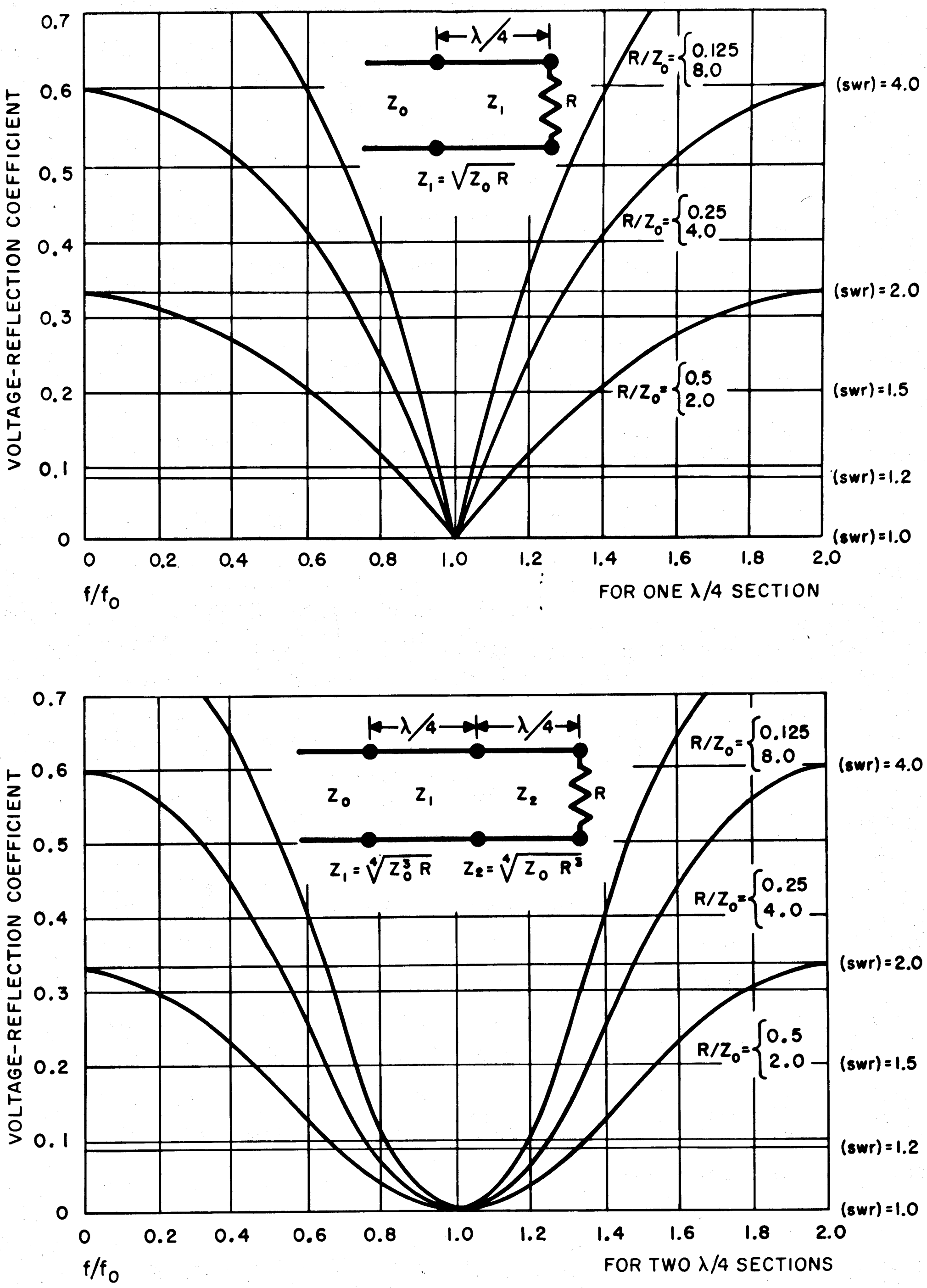


Fig. 17—Quarter-wave matching sections.

In either lumped or distributed resonators

(dissipation loss)

$$= 10 \log_{10}(P_x/P_{\text{out}})$$

$$= 10 \log_{10}[1/(1-Q_{1s}/Q_{1u})(1-Q_{2s}/Q_{2u})]$$

$$\approx 20 \log_{10}[1/(1-Q_s/Q_u)]$$

$$\approx 20 \log_{10}(1+Q_s/Q_u)$$

$$\approx 8.7 Q_s/Q_u \text{ decibels}$$

where

$$Q_s/Q_u = [(Q_{1s}/Q_{1u})(Q_{2s}/Q_{2u})]^{1/2}$$

provided (Q_{1s}/Q_{1u}) and (Q_{2s}/Q_{2u}) do not differ by a ratio of more than 4 to 1, and neither exceeds 0.2.

(mismatch loss at f_0)

$$= 10 \log_{10}(P_m/P_x)$$

$$= 10 \log_{10}[(1+p)^2/4p] \text{ decibels.}$$

Equations and curves for selectivity are given on pp. 9–6, 9–7, and 9–9, where $Q = Q_s$.

At the peaks, when $p \geq 1$, the mismatch loss is zero, except for some that is included in the dissipation loss.

Input voltage standing-wave ratio at f_0 for equal or unequal resonators

$$S = \frac{p + Q_{1s}/Q_{1u}}{1 - Q_{1s}/Q_{1u}}.$$

At the peak frequencies ($p \geq 1$) for equal or nearly equal resonators

$$S = \frac{1 + Q_{1s}/Q_{1u}}{1 - Q_{1s}/Q_{1u}}.$$

Similarly at the output, using subscript 2 instead of 1.

When the resonators are isolated, each one presents to the generator or load an swr of

$$S = (Q_u/Q_s) - 1.$$

The power dissipation in either lumped or distributed (quarter-wave) devices, where the two resonators are not necessarily identical, but $Q_s \ll Q_u$, is

$$P_{1c} = I_{1sc}{}^2 R_{1u}[4/(1+p)^2] P_m Q_{1s}/Q_{1u}$$

$$P_{2c} = [4p/(1+p)^2] P_m Q_{2s}/Q_{2u}.$$

These equations and those below for the currents assume that P_m is concentrated at f_0.

The currents in quarter-wave resonant lines, when $Q_s \ll Q_u$

$$I_{1sc} = [4/(1+p)](P_m Q_{1s}/\pi Z_{10})^{1/2}$$

$$I_{2sc}/I_{1sc} = (pZ_{10}Q_{2s}/Z_{20}Q_{1s})^{1/2}.$$

Similarly, for a pair of tuned circuits at resonance, when $Q_s \ll Q_u$

$$I_1 = [2/(1+p)](P_m Q_{1s}/X_1)^{1/2}$$

$$I_2/I_1 = (pX_1Q_{2s}/X_2Q_{1s})^{1/2}.$$

QUARTER-WAVE MATCHING SECTIONS

Figure 17 shows how voltage-reflection coefficient and standing-wave ratio (swr) vary with frequency f when quarter-wave matching lines are inserted between a line of characteristic impedance Z_0 and a load of resistance R. f_0 is the frequency for which the matching sections are exactly one-quarter wavelength ($\lambda/4$) long.

IMPEDANCE MATCHING WITH SHORTED STUB *(Fig. 18)*

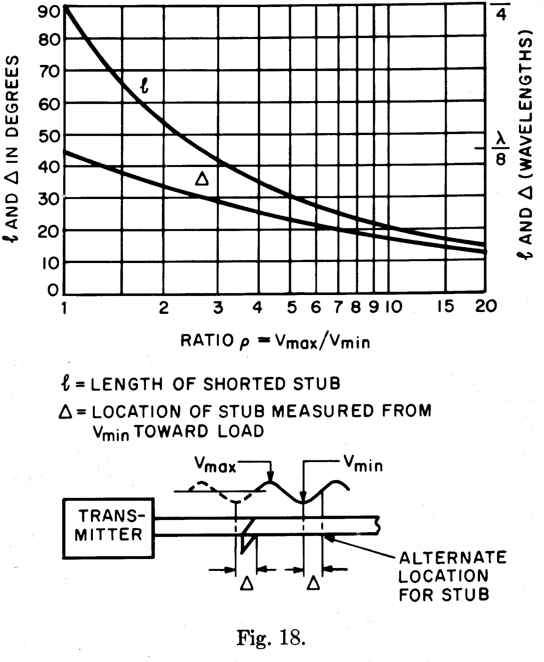


Fig. 18.

IMPEDANCE MATCHING WITH OPEN STUB *(Fig. 19)*

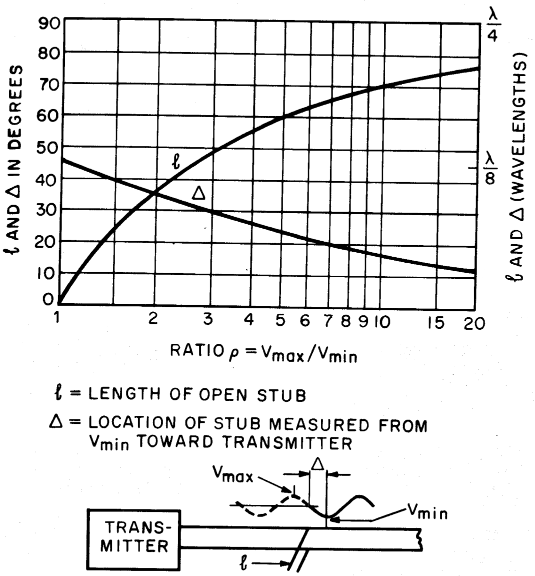


Fig. 19.

LENGTH OF TRANSMISSION LINE

Figure 20 gives the actual length of line in centimeters and inches when given the length in electrical degrees and the frequency, provided that the velocity of propagation on the transmission line is equal to that in free space. The length is equal to that in free space. The length is given on the L-scale intersection by a line between λ and $l°$, where

$$l° = \frac{360L \text{ in centimeters}}{\lambda \text{ in centimeters}}.$$

Example: $f = 600$ megahertz, $l° = 30$, length $L =$ 1.64 inches or 4.2 centimeters.

MEASUREMENT OF IMPEDANCE WITH SLOTTED LINE

Symbols

Z_0 = characteristic impedance of line
Z = impedance of load (the unknown)
Z_1 = impedance at first $V_{\min}$
λ = wavelength on line
χ = distance from load to first $V_{\min}$
$(\text{swr}) = V_{\max}/V_{\min}$
$\theta° = 180\,(\chi/\tfrac{1}{2}\lambda) = 0.012 f\chi/k$
k = velocity factor
$= (\text{velocity on line})/(\text{velocity in free space})$

where f is in megahertz and χ in centimeters.

Procedure (Fig. 21)

Measure $\lambda/2$, χ, $V_{\max}$, and $V_{\min}$.
Determine

$$Z_1/Z_0 = 1/(\text{swr}) = V_{\min}/V_{\max}$$

$$(\text{wavelengths toward load}) = \chi/\lambda = 0.5\chi/(\lambda/2).$$

Then Z/Z_0 may be found on an impedance chart. For example, suppose

$$V_{\min}/V_{\max} = 0.60 \quad \text{and} \quad \chi/\lambda = 0.40.$$

Refer to the chart, such as Fig. 22 reproduced in part here. Lay off with slider or dividers the distance on the R/Z_0 axis from the center point (marked 1.0) to 0.60. Pass around the circumference of the chart in a counterclockwise direction from the starting point 0 to the position 0.40, toward the load. Read off the resistance and reactance components of the normalized load impedance Z/Z_0 at the point of the dividers. Then it is found that

$$Z = Z_0(0.77 + j0.39).$$

Similarly, there may be found the admittance of the load. Determine

$$Y_1/Y_0 = V_{\max}/V_{\min} = 1.67$$

in the above example. Now pass around the chart counterclockwise through $\chi/\lambda = 0.40$, starting at 0.25 and ending at 0.15. Read the components of the normalized admittance.

$$Y = 1/Z = (1/Z_0)(1.03 - j0.53).$$

Alternatively, these results may be computed by

$$\begin{aligned}
Z &= R_s + jX_s \\
&= Z_0 \frac{1 - j(\text{swr})\tan\theta}{(\text{swr}) - j\tan\theta} \\
&= Z_0 \frac{2(\text{swr}) - j[(\text{swr})^2 - 1]\sin 2\theta}{[(\text{swr})^2 + 1] + [(\text{swr})^2 - 1]\cos 2\theta}
\end{aligned}$$

$$\begin{aligned}
Y &= G + jB = 1/Z = (1/R_p) - j(1/X_p) \\
&= Y_0 \frac{2(\text{swr}) + j[(\text{swr})^2 - 1]\sin 2\theta}{[(\text{swr})^2 + 1] - [(\text{swr})^2 - 1]\cos 2\theta}
\end{aligned}$$

where R_s and X_s are the series components of Z, while R_p and X_p are the parallel components.

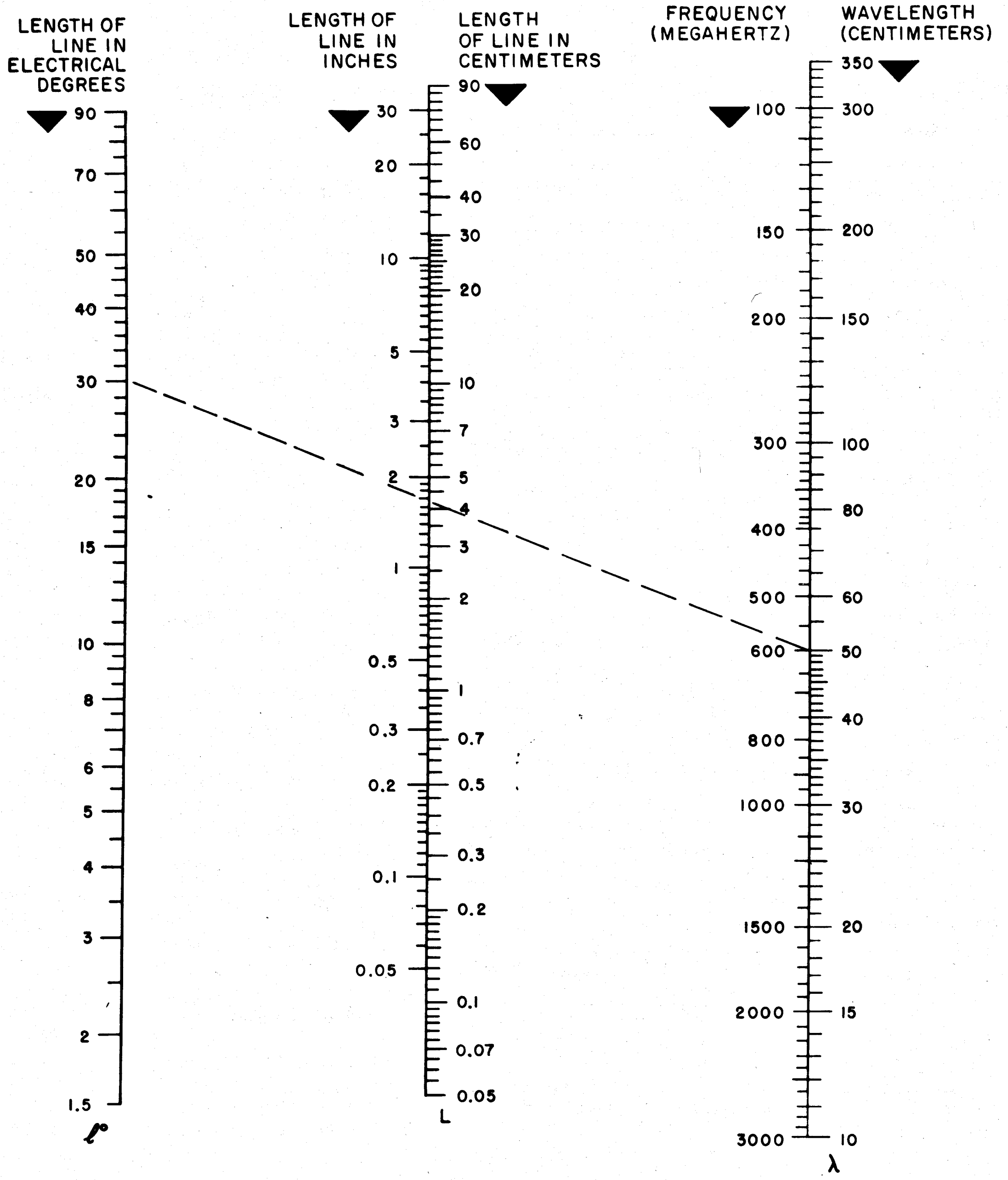


Fig. 20—Determination of length of transmission line.

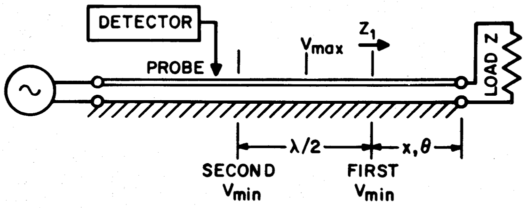


Fig. 21—Measurements on line.

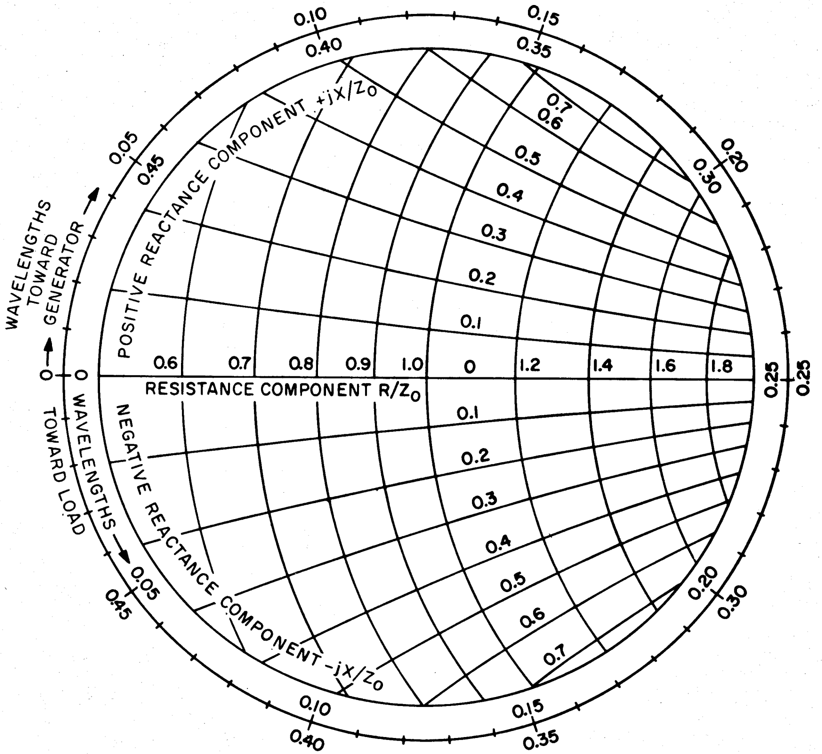
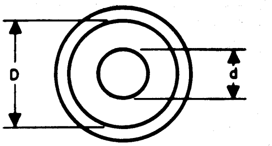


Fig. 22—Smith chart, center portion.

CHARACTERISTIC IMPEDANCE OF LINES

A. Single coaxial line (See also Fig. 23).



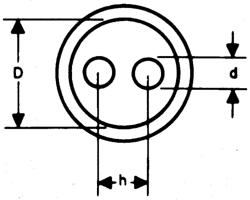
$$Z_0 = (138/\epsilon^{1/2}) \log_{10} (D/d)$$

$$= (60/\epsilon^{1/2}) \log_e (D/d)$$

ϵ = dielectric constant

= 1 in air

B. Balanced shielded line



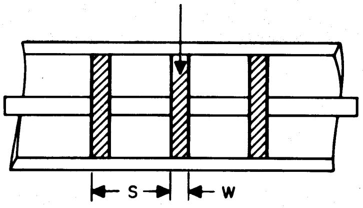
For $D \gg d$, $h \gg d$,

$$Z_0 = (276/\epsilon^{1/2}) \log_{10} \{2v[(1-\sigma^2)/(1+\sigma^2)]\}$$

$$= (120/\epsilon^{1/2}) \log_e \{2v[(1-\sigma^2)/(1+\sigma^2)]\}$$

$$v = h/d \qquad \sigma = h/D$$

C. Beads—dielectric ϵ_1

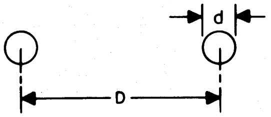


For lines *A*. and *B*., if insulating beads are used at frequent intervals—call new characteristic impedance Z_0'

$$Z_0' = Z_0/\{1+[(\epsilon_1/\epsilon)-1](W/S)\}^{1/2}$$

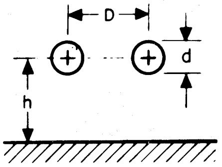
$$W \ll S \ll \lambda/4$$

D. Open 2-wire line in air (See also Fig. 23).



$$Z_0 = 120 \cosh^{-1}(D/d)$$
$$\approx 276 \log_{10}(2D/d)$$
$$\approx 120 \log_e(2D/d)$$

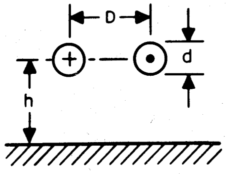
E. Wires in parallel, near ground



For $d \ll D, h$,

$$Z_0 = (69/\epsilon^{1/2}) \log_{10}\{(4h/d)[1+(2h/D)^2]^{1/2}\}$$

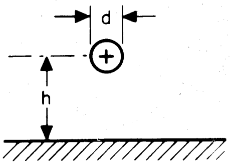
F. Balanced, near ground



For $d \ll D, h$,

$$Z_0 = (276/\epsilon^{1/2}) \log_{10}\{(2D/d)[1+(D/2h)^2]^{-1/2}\}$$

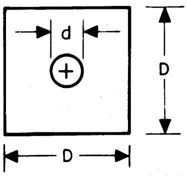
G. Single wire, near ground



For $d \ll h$,

$$Z_0 = (138/\epsilon^{1/2}) \log_{10}(4h/d)$$

H. Single wire, square enclosure

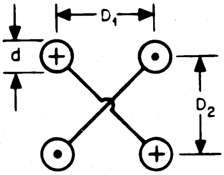


$$Z_0 \approx [138 \log_{10}\rho + 6.48 - 2.34A - 0.48B - 0.12C]\epsilon^{-1/2}$$

where $\rho = D/d$

$$A = (1+0.405\rho^{-4})/(1-0.405\rho^{-4})$$
$$B = (1+0.163\rho^{-8})/(1-0.163\rho^{-8})$$
$$C = (1+0.067\rho^{-12})/(1-0.067\rho^{-12})$$

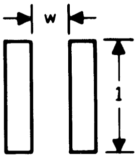
I. Balanced 4-wire



For $d \ll D_1, D_2$,

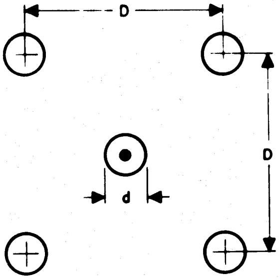
$$Z_0 = (138/\epsilon^{1/2}) \log_{10}\{(2D_2/d)[1+(D_2/D_1)^2]^{-1/2}$$

J. Parallel-strip line



$$w/l < 0.1$$
$$Z_0 \approx 377(w/l)$$

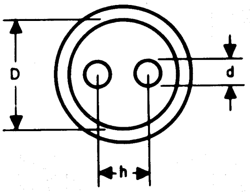
K. Five-wire line



For $d \ll D$,

$$Z_0 = (173/\epsilon^{1/2}) \log_{10}(D/0.933d)$$

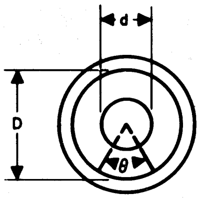
L. Wires in parallel—sheath return



For $d \ll D, h$,

$$Z_0 = (69/\epsilon^{1/2}) \log_{10}[(\nu/2\sigma^2)(1-\sigma^4)]$$
$$\sigma = h/D$$
$$\nu = h/d$$

M. Air coaxial with dielectric supporting wedge

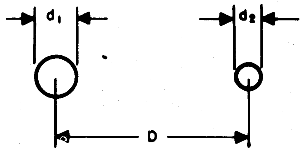


$$Z_0 \approx \frac{138 \log_{10}(D/d)}{[1+(\epsilon-1)(\theta/360)]^{1/2}}$$

ϵ = dielectric constant of wedge

θ = wedge angle in degrees

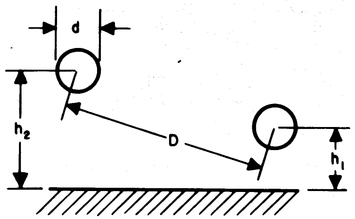
N. Balanced 2-wire—unequal diameters



$$Z_0 = (60/\epsilon^{1/2}) \cosh^{-1} N$$

$$N = \tfrac{1}{2}[(4D^2/d_1d_2) - (d_1/d_2) - (d_2/d_1)]$$

O. Balanced 2-wire near ground



For $d \ll D, h_1, h_2$,

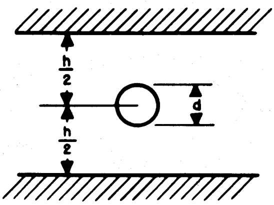
$$Z = (276/\epsilon^{1/2}) \log_{10}\{(2D/d)[1+(D^2/4h_1h_2)]^{-1/2}\}$$

Holds also in either of the following special cases:

$$D = \pm(h_2 - h_1)$$

$$h_1 = h_2 \text{ (see } F\text{. above)}$$

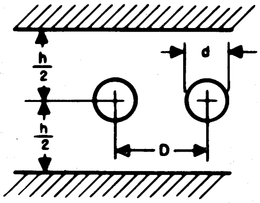
P. Single wire between grounded parallel planes—ground return



For $d/h < 0.75$,

$$Z_0 = (138/\epsilon^{1/2}) \log_{10}(4h/\pi d)$$

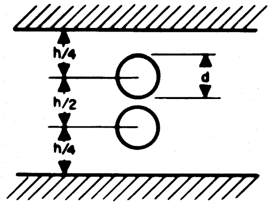
Q. Balanced line between grounded parallel planes



For $d \ll D, h$,

$$Z_0 = (276/\epsilon^{1/2}) \log_{10}\left(\frac{4h \tanh(\pi D/2h)}{\pi d}\right)$$

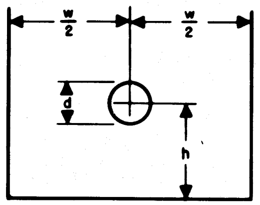
R. Balanced line between grounded parallel planes



For $d \ll h$,

$$Z_0 = (276/\epsilon^{1/2}) \log_{10}(2h/\pi d)$$

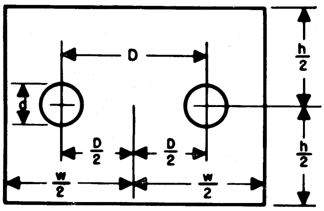
S. Single wire in trough



For $d \ll h, w$,

$$Z_0 = (138/\epsilon^{1/2}) \log_{10}\left(\frac{4w \tanh(\pi h/w)}{\pi d}\right)$$

T. Balanced 2-wire line in rectangular enclosure



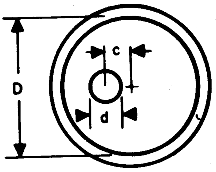
For $d \ll D, w, h$,

$$Z_0 = (276/\epsilon^{1/2}) \left[\log_{10}\left(\frac{4h \tanh(\pi D/2h)}{\pi d}\right) - \sum_{m=1}^{\infty} \log_{10}\left(\frac{1+u_m{}^2}{1-v_m{}^2}\right)\right]$$

where

$$u_m = \frac{\sinh(\pi D/2h)}{\cosh(m\pi w/2h)} \qquad v_m = \frac{\sinh(\pi D/2h)}{\sinh(m\pi w/2h)}$$

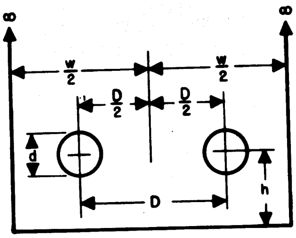
U. Eccentric line



$$Z_0=(60/\epsilon^{1/2})\cosh^{-1}U$$

$$U=\tfrac{1}{2}[(D/d)+(d/D)-(4c^2/dD)]$$

V. Balanced 2-wire line in semi-infinite enclosure



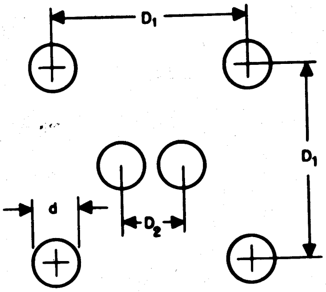
For $d \ll D, w, h$,

$$Z_0=(276/\epsilon^{1/2})\log_{10}[2w/\pi d(A^{1/2})]$$

where

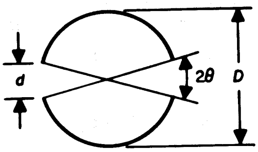
$$A=\operatorname{cosec}^2(\pi D/w)+\operatorname{cosech}^2(2\pi h/w)$$

W. Outer wires grounded, inner wires balanced to ground



$$Z_0\approx(276/\epsilon^{1/2})\left\{\log_{10}(2D_2/d)-\left[\log_{10}\frac{1+(1+D_2/D_1)^2}{1+(1-D_2/D_1)^2}\right]^2\left[\log_{10}(2D_1\sqrt{2}/d)\right]^{-1}\right\}$$

X. Split thin-walled cylinder



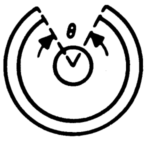
$$Z_0\approx\frac{129}{\log_{10}[\cot\frac{1}{2}\theta+(\cot^2\frac{1}{2}\theta-1)^{1/2}]}$$

For θ small:

$$Z_0\approx129/\log_{10}(4D/d)$$

Courtesy of Electronic Engineering

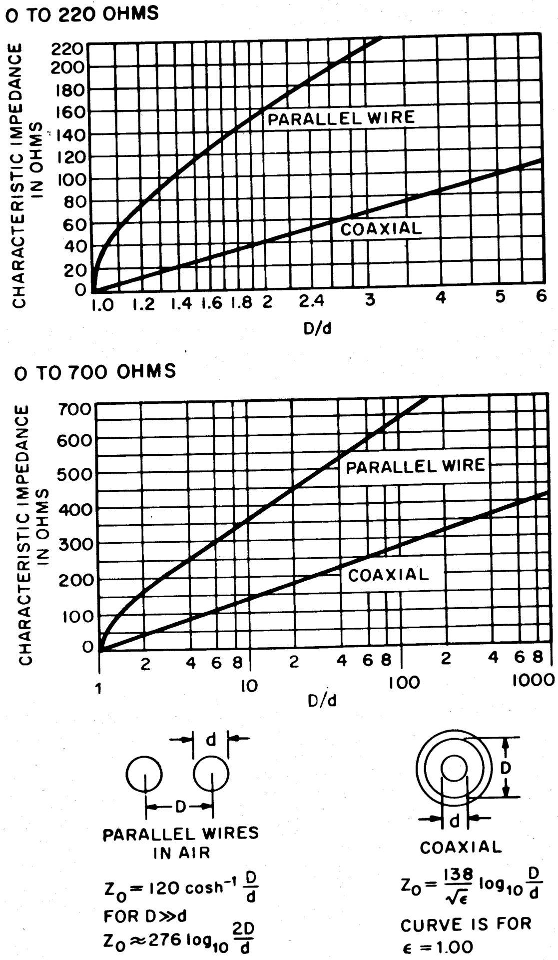
Y. Slotted air line



When a slot is introduced into an air coaxial line for measuring purposes, the increase in characteristic impedance in ohms, compared with a normal coaxial line, is less than

$$\Delta Z=0.03\theta^2$$

where θ is the angular opening of the slot in radians.



PARALLEL WIRES IN AIR

$Z_0 = 120 \cosh^{-1}\frac{D}{d}$

FOR $D \gg d$

$Z_0 \approx 276 \log_{10}\frac{2D}{d}$

COAXIAL

$Z_0 = \frac{138}{\sqrt{\epsilon}}\log_{10}\frac{D}{d}$

CURVE IS FOR $\epsilon = 1.00$

Fig. 23.

VOLTAGE GRADIENT IN A COAXIAL LINE *(Fig. 24)*

C'= capacitance in picofarads/foot

D= diameter of inner surface of outer conductor in same units as d

d= diameter of inner conductor

E= total voltage across line (E and ΔE both rms or both peak)

r= radius (r and Δr both in same units)

ϵ= net effective dielectric constant (=1 for air); $1/\epsilon^{1/2}$= velocity factor

$$\Delta E/\Delta r = \frac{0.434E}{r \log_{10}(D/d)}$$

$$= 0.059EC'/r\epsilon$$

$$= 60E/rZ_0\epsilon^{1/2}$$

$$= (6.10 \times 10^4 E)/rZ_0^2C'.$$

At the voltage standing-wave maximum

(gradient at surface of inner conductor)

$$= (5.37/d)(SP_{\mathrm{kW}}/Z_0\epsilon)^{1/2}$$

$$= 5450(SP_{\mathrm{kW}})^{1/2}/dC'Z_0^{3/2} \text{ peak volts/mil}$$

where d is in inches (1 mil=0.001 inch). For amplitude or pulse modulation, let P_{kW} be the power in kilowatts at the crest of the modulation cycle. Thus, if the carrier is 1 kilowatt and modulation 100 percent, set

$$P_{\mathrm{kW}} = 4 \text{ kilowatts.}$$

Example: What is the voltage gradient at inner conductor of a $6\frac{1}{8}$-inch rigid 50-ohm line with 500 kilowatts continuous-wave power, unity swr? Let $\epsilon = 1.00$ and $d = 2.60$ inches.

$$(\text{gradient}) = (5.37/2.60)(500/50)^{1/2}$$

$$= 6.55 \text{ peak volts/mil.}$$

The breakdown strength of air at atmospheric pressure is 29 000 peak volts/centimeter, or 74 peak volts/mil (experimental value, before derating).

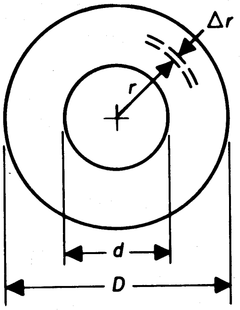


Fig. 24.

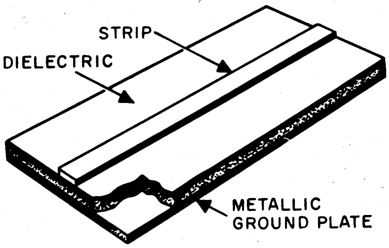


Fig. 25—Microstrip line.

MICROSTRIP*

Microstrip consists of a wire above a ground plane, being analogous to a 2-wire line in which one of the wires is represented by the image in the ground plane of the wire that is physically present. Figure 25 illustrates a short length of microstrip line, showing the metallic-strip conductor bonded to a dielectric sheet, to the other side of which is bonded a metallic ground plate.

Phase Velocity

Theoretically, for the TEM mode with conductors completely immersed in the dielectric, the velocity of propagation is

$$v = c/(\epsilon_r)^{1/2}$$

where c= velocity of light in vacuum and ϵ_r= dielectric constant relative to air.

For Teflon-impregnated Fibreglas dielectric, this gives 604 feet per microsecond. Experimental measurements on a line with $7/32$-inch strip width and dielectric sheet $1/16$-inch thick give

$$v = 655 \text{ feet/microsecond.}$$

Typical measurements together with the theoretical TEM wavelength are plotted in Fig. 26.

Characteristic Impedance

If it were not for fringing and leakage flux, the theoretical characteristic impedance would be

$$Z_0 = (h/w)(\mu/\epsilon)^{1/2}$$

$$= 377(h/w)(1/\epsilon_r)^{1/2}$$

---

* D. D. Grieg and H. F. Engelmann, "Microstrip—A New Transmission Technique for the Kilomegacycle Range," and two accompanying papers in *Proceedings of the IRE*, vol. 40, pp. 1644–1663; December 1952; also *Electrical Communication*, vol. 30, pp. 26–54; March 1953.

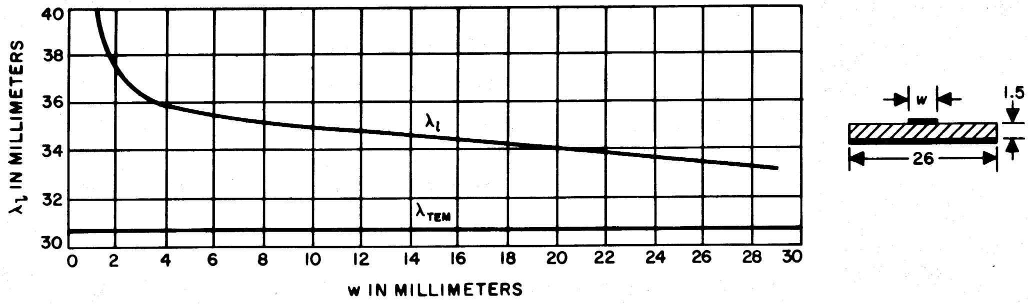


Fig. 26—Wavelength in microstrip versus width of strip conductor. The dimensions in the sketch at right are in millimeters. Dielectric was Fibreglas G–6. Measurements were taken at 4770 megahertz.

where h= thickness of dielectric, w= width of strip conductor, ϵ= dielectric constant in farads/meter, and μ= permeability in henries/meter.

Figure 27 shows the experimentally determined Z_0 for typical microstrip lines.

Attenuation

Conductor loss for copper, in decibels/foot

$$\alpha_{\mathrm{Cu}} = 7.25 \times 10^{-5} (1/h) (f_{\mathrm{MHz}} \epsilon_r)^{1/2}.$$

Dielectric loss in decibels/foot

$$\alpha_d = 2.78 \times 10^{-2} f_{\mathrm{MHz}} F_p (\epsilon_r)^{1/2}$$

where F_p= power factor or loss angle, and h= dielectric thickness in inches.

A correction factor for conductor attenuation is shown in Fig. 28 for use in the equation

$$\alpha_c = \alpha_0 \times \Delta$$

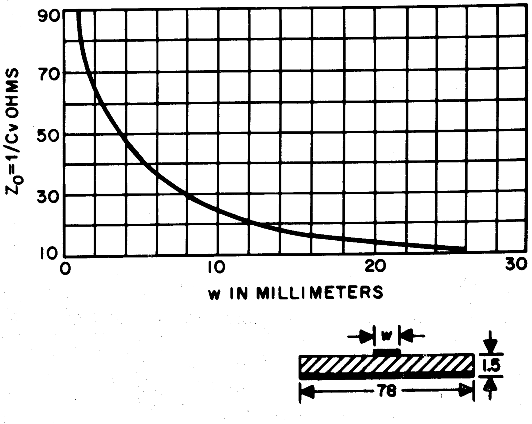


Fig. 27—Characteristic impedance for microstrip with Fibreglas G–6 dielectric. Dimensions in sketch are in millimeters. C is the measured electrostatic capacitance in farads per unit length and v is the phase velocity in units of length per second.

where α_0 is, for copper conductors, given by α_{Cu} above.

$$\alpha_0 = \alpha_{\mathrm{Cu}} (\mu_r \rho / \rho_{\mathrm{Cu}})^{1/2}$$

where μ_r= relative permeability, and ρ/ρ_{Cu}= resistivity relative to copper.

The measured attenuation of a typical microstrip line is shown in Fig. 40. The relatively high attenuation is due to the small physical size of the line.

Power-Handling Capacity

For a microstrip line composed of a strip 7/32-inch wide on a Teflon-impregnated Fibreglas base 1/16-inch thick:

(**A**) At 3000 megahertz with 300 watts cw, the temperature under the strip conductor has been measured at 50° Celsius rise above 20° Celsius ambient.

(**B**) Under pulse conditions, corona effects appear at the edge of the strip conductor for pulse power of roughly 10 kilowatts at 9000 megahertz.

STRIP TRANSMISSION LINES*

Strip transmission lines differ from microstrip in that a second ground plane is placed above the conductor strip (Fig. 29). The characteristic impedance is shown in Fig. 30 and the attenuation in Fig. 31.

Attenuation

Dielectric loss in decibels/unit length

$$\alpha_d = 27.3 F_p (\epsilon_r)^{1/2} / \lambda_0$$

where λ_0= free-space wavelength.

---

* S. B. Cohn, "Problems in Strip Transmission Lines," *Transactions of the IRE Professional Group on Microwave Theory and Techniques*, vol. MTT3, pp. 119–126; March 1955. Other papers on strip-type lines also appear in that issue of the journal.

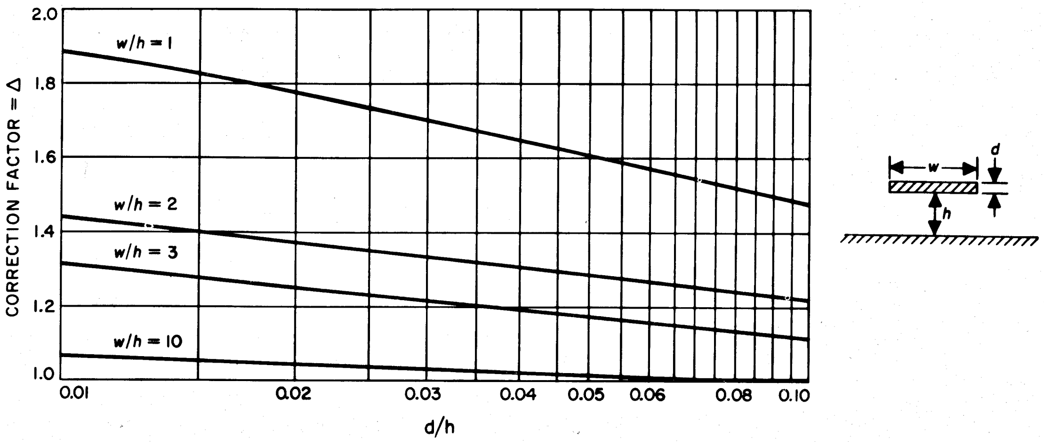


Fig. 28—Correction factor Δ for conductor attenuation (case of wide strip of small thickness above infinite ground plane).

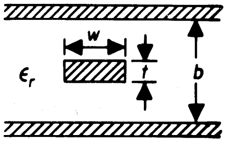


Fig. 29.

Conductor loss in decibels/unit length

$$\alpha_c = (y/b)\,(f_{\mathrm{GHz}}\epsilon_r\mu_r\rho/\rho_{\mathrm{Cu}})^{1/2}$$

where y = ordinate from Fig. 31, and ρ/ρ_{Cu} = resistivity relative to copper.

The unit of length in α_d is that of λ_0, and in α_c it is that of b.

LINES WITH HELICAL INNER CONDUCTOR

Spiral Delay Line

For a transmission line with helical inner conductor (spiral delay line) where axial wavelength and length of line are both long compared with line diameter (similar to Fig. 32 in dimensional symbols):

$$L' = 0.30n^2d^2[1-(d/D)^2]$$

microhenries/axial foot, where d is in inches and $n = 1/\tau$ = turns/inch.

$$C' = 7.4\epsilon_r/\log_{10}(D/d)$$

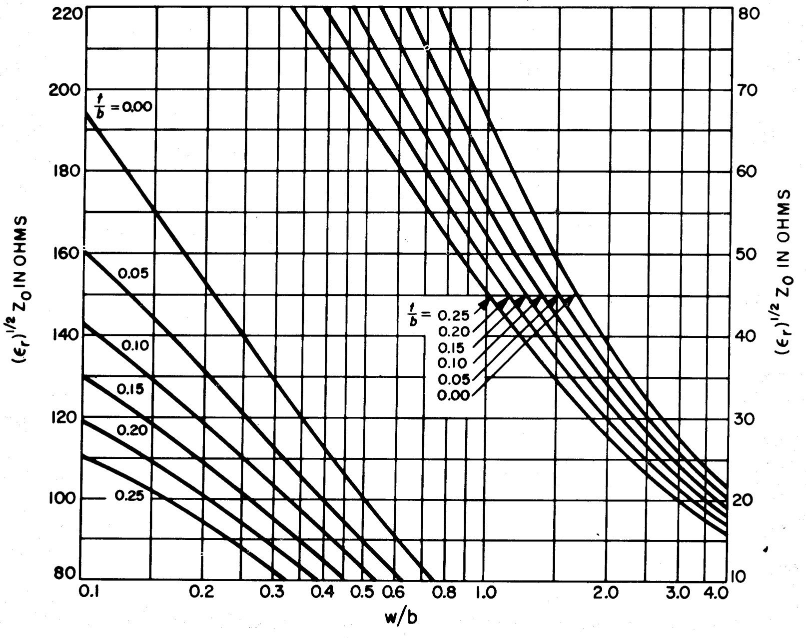


Fig. 30—Plot of strip-transmission-line Z_0 versus w/b for various values of t/b. For lower-left family of curves, refer to left-hand ordinate values; for upper-right curves, use right-hand scale. *Courtesy of Transactions of the IRE Professional Group on Microwave Theory and Techniques.*

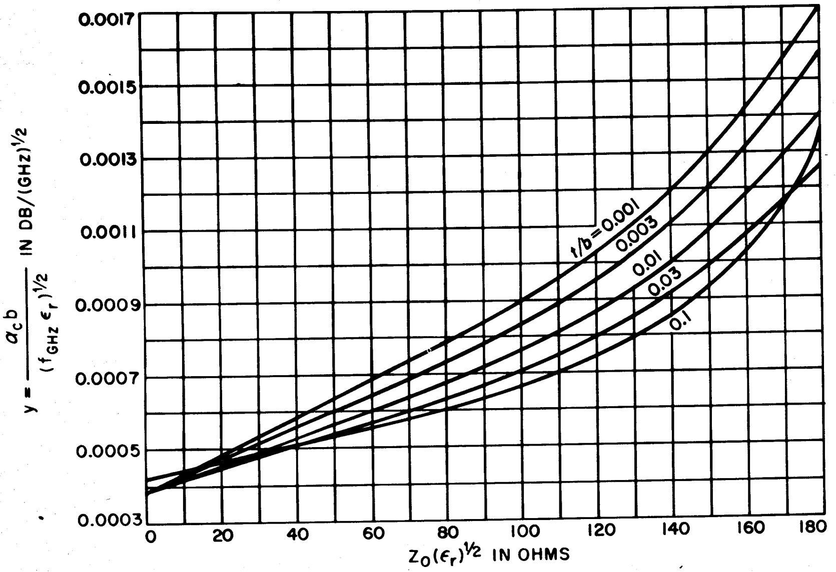


Fig. 31—Theoretical attenuation of copper-shielded strip transmission line in dielectric medium ϵ_r. *Courtesy of Transactions of the IRE Professional Group on Microwave Theory and Techniques.*

picofarads/axial foot.

$$Z_0 = (L'/C')^{1/2} \times 10^3 \text{ ohms}$$

$$T = (L'C')^{1/2} \times 10^{-3}$$

microseconds/axial foot.

$$\alpha_{\text{dB}} = 4.34R/Z_0 + 27.3F_p fT$$

decibels/axial foot where R= total conductor resistance in ohms/axial foot, f= frequency in megahertz, F_p= power factor, and ϵ_r= relative dielectric constant of medium between spiral and outer conductor.

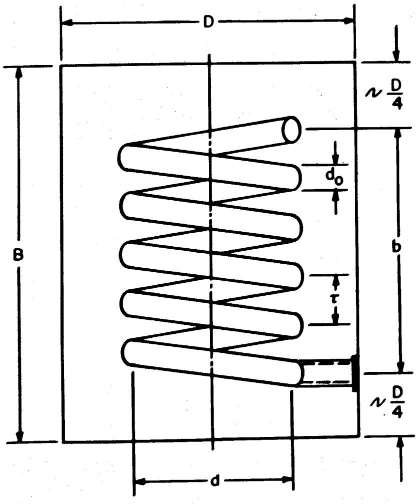


Fig. 32—Outline sketch of resonator. *W. W. Macalpine and R. O. Schildknecht, "Coaxial Resonators with Helical Inner Conductor," Proceedings of the IRE, vol. 47, no. 12, p. 2100; December 1959. © 1959 Institute of Radio Engineers.*

HELICAL RESONATOR*

Symbols

b= axial length of coil, inches
B= inside length of shield, inches
d= mean diameter of turns, inches
D= inside diameter of shield, inches
d_0= diameter of conductor, inches
f_0= frequency of resonance, megahertz
n= turns per inch
N= total number of turns of winding
Q_u= unloaded Q

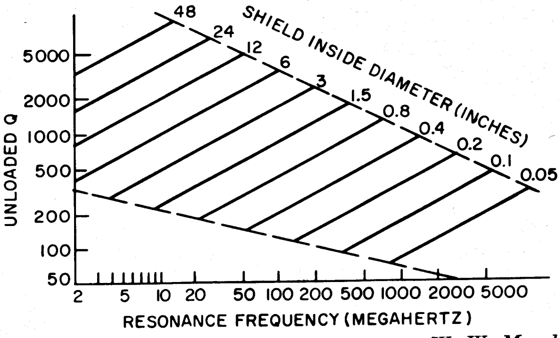


Fig. 33—Unloaded Q of helical resonator. *W. W. Macalpine and R. O. Schildknecht, "Coaxial Resonators with Helical Inner Conductor," Proceedings of the IRE, vol. 47, no. 12, p. 2100; December 1959. © 1959 Institute of Radio Engineers.*

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* W. W. Macalpine and R. O. Schildknecht, "Coaxial Resonators with Helical Inner Conductor," *Proceedings of the IRE*, vol. 47, no. 12, pp. 2099–2105; December 1959. W. W. Macalpine and R. O. Schildknecht, "Helical Resonator Design Chart," *Electronics*, p. 140; 12 August 1960.

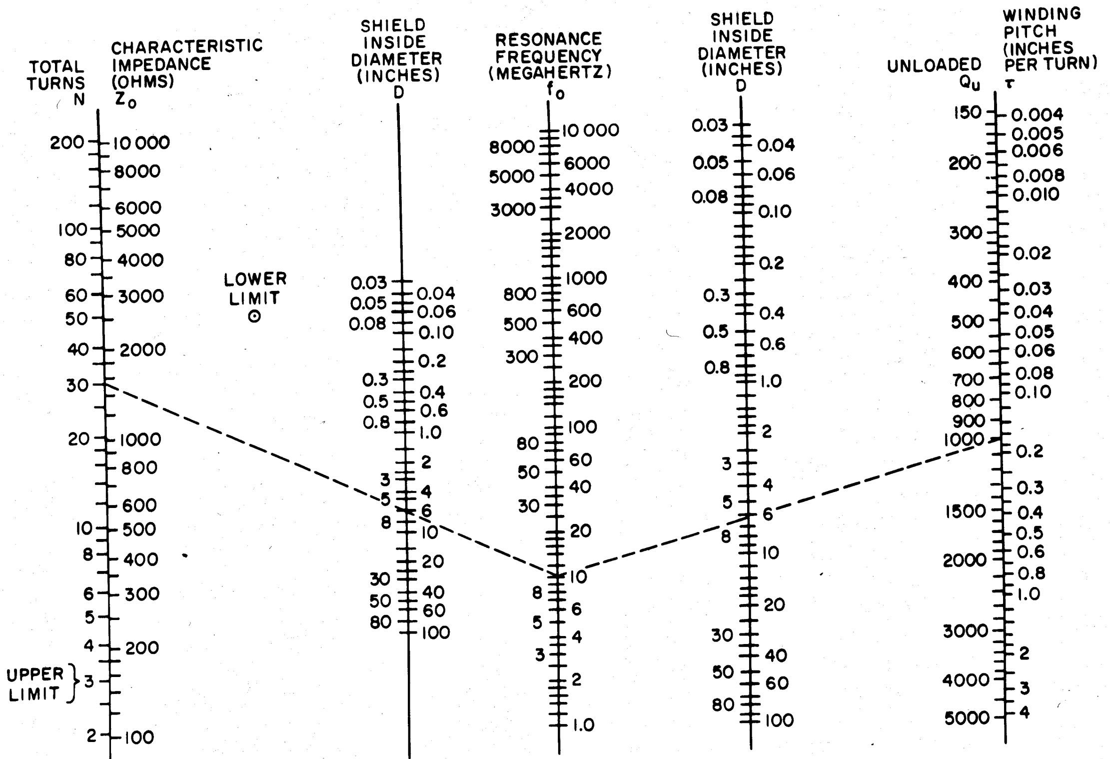

Fig. 34—Design chart for quarter-wave helical resonators. *W. W. Macalpine and R. O. Schildknecht, "Coaxial Resonators with Helical Inner Conductor," Proceedings of the IRE, vol. 47, no. 12, p. 2101; December 1959.* © *1959 Institute of Radio Engineers.*

δ = skin depth, inch

$\tau = 1/n$ = pitch of winding, inches.

Other symbols have the usual meanings, page 24-1.

The helical resonator shown in Fig. 32 consists of a shield (outer conductor) enclosing a coil (inner conductor). One end of the coil is solidly connected to the shield and the other end open-circuited, except possibly for a trimming capacitor. It operates as a distributed-parameter system equivalent to a quarter-wave coaxial transmission-line resonator. Probe, loop, or aperture coupling can be used for input and output circuits, or between adjacent coupled resonators.

Unloaded Q versus resonance frequency and shield diameter is shown in Fig. 33. The region plotted is where the helical resonator gives better Q for a given volume than other types. For higher Q and frequency, a conventional coaxial-line resonator is preferable. When the Q and frequency are lower than the plotted region, a lumped LC resonant circuit is indicated. These conditions are marked on Fig. 34 as "upper limit" (3 turns) and "lower limit," respectively.

Figure 34 is usually accurate to within ±10 percent. It is plotted from the following equations and is limited by the conditions noted.

Unloaded Q is given for a resonator that consists of a single-layer coil of copper conductor on a low-loss form and is enclosed in a shield of seamless copper tubing. A shielded coil below its resonance frequency also has ideally a true Q predicted by this equation. The equation gives a practical working value of Q somewhat below the theoretical maximum.

$$Q_u = 50 D f_0^{1/2}$$

provided:

$$0.45 < d/D < 0.6$$

$$b/d > 1.0$$

$$0.4 < d_0/\tau < 0.6 \text{ at } b/d = 1.5$$

$$0.5 < d_0/\tau < 0.7 \text{ at } b/d = 4.0$$

$$d_0 > 5\delta.$$

Total number of turns

$$N = 1900/(f_0 D) \text{ turns}$$

for $d/D = 0.55$, and $b/d > 1.0$.

Pitch of winding and characteristic impedance:

$$\tau = 1/n = (f_0 D^2)/2300 \text{ inches per turn}$$

$$Z_0 = 98\,000/(f_0 D) \text{ ohms}$$

for $d/D = 0.55$, and $b/d = 1.5$.

General conditions for all equations:

$$B \approx (b + D/2)$$

$$\tau < d/2.$$

Simplified and empirical equations from which

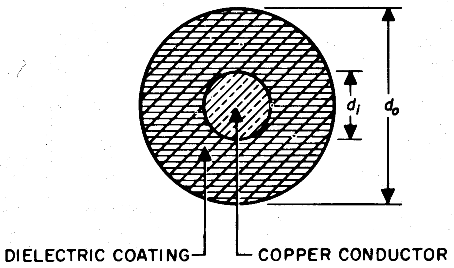

Fig. 35—Cross section of surface-wave transmission line.

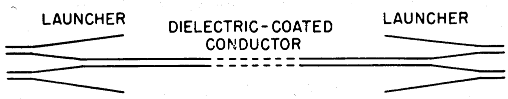

Fig. 36—Surface-wave transmission line with launchers at each end. These form transitions to coaxial line. *Courtesy of Electronics.*

the design equations are developed:

$$L=0.025n^2d^2[1-(d/D)^2] \text{ microhenry per axial inc}$$

$$C=0.75/\log_{10}(D/d) \text{ picofarad per axial inch}$$

$$v=f_0\lambda=1000(LC)^{-1/2} \text{ inches per microsecond}$$

$$b=0.94\lambda/4=235f_0^{-1}(LC)^{-1/2} \text{ inches}$$

$$Z_0=1000(L/C)^{1/2}=235\,000(bf_0C)^{-1} \text{ ohms.}$$

A further useful relationship in terms of the inside volume of the shield (vol) in cubic inches i

$$Q_u=50(\text{vol})^{1/3}f_0^{1/2}$$

provided that $0.4<d/D<0.6$, and $1.0<b/d<3.0$.

SURFACE-WAVE TRANSMISSION LINE*

The surface-wave transmission line is a single-conductor line having a relatively thick dielectric

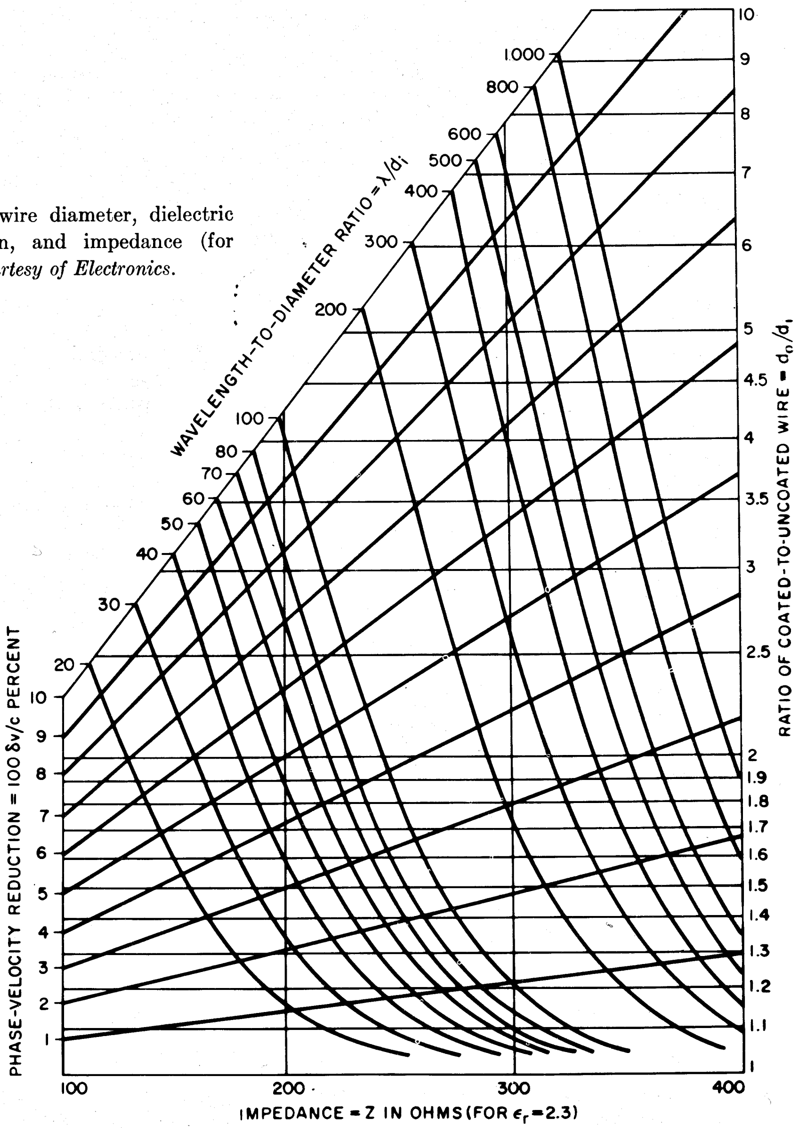

Fig. 37—Relationship among wire diameter, dielectric layer, phase-velocity reduction, and impedance (for brown polyethylene). *Courtesy of Electronics.*

* G. Goubau, "Designing Surface-Wave Transmission Lines," *Electronics*, vol. 27, pp. 180–184; April 1954.

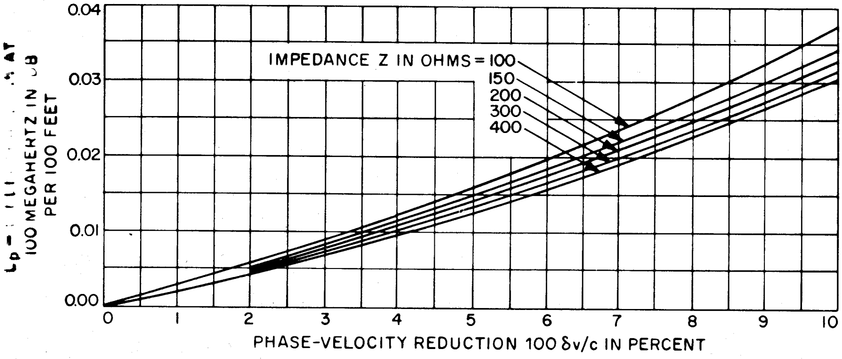

Fig. 38—Dielectric loss at 100 megahertz for brown polyethylene ($\epsilon_r = 2.3$ and $F_p = 5 \times 10^{-4}$). *Courtesy of Electronics.*

sheath (Fig. 35). The sheath diameter is often 3 or more times the conductor diameter. A mode of propagation that is practically nonradiating is excited on the line by means of a conical horn at each end as shown in Fig. 36. The mouth of the horn is roughly one-quarter to one wavelength in diameter. Losses are about half those of a 2-wire line, but the surface-wave line has a practical lower frequency limit of about 50 megahertz.

Design charts are given in Figs. 37 to 39 together with equations for attenuation losses.

The losses in the two launchers combined vary from less than 0.5 decibel to a little more than 1.0 decibel, according to their design.

Conductor loss L_c by the equation below is 5 percent over the theoretical value for pure copper. Dielectric loss L_p for polyethylene at 100 megahertz is shown in Fig. 38. For other dielectrics and frequencies, find L_i by the equation.

$L_c = 0.455 f^{1/2}/Zd_i$ decibels/100 feet

$L_i = 26 f F_p L_p / (\epsilon_r - 1)$ decibels/100 feet

$L_i = L_p f/100$

for brown polyethylene (Fig. 38).

Symbols

c = velocity of propagation in free space
d_i = diameter of the conductor (inches in equation for L_c)
d_o = outside diameter of the dielectric coating
f = frequency in megahertz
F_p = power factor of dielectric
L_c = conductor loss in decibels/100 feet
L_i = dielectric loss in decibels/100 feet
L_p = dielectric loss shown in Fig. 38
Z = waveguide impedance in ohms
δv = reduction in phase velocity
ϵ_r = dielectric constant relative to air
λ = free-space wavelength.

Example: At 900 megahertz ($\lambda = 0.333$ meter), a 200-foot line is required having a permissible loss of 1.0 decibel/100 feet (not including the launcher losses). What are its dimensions?

Allowing 20 percent for dielectric loss, the conductor loss would be $L_c = 0.8$ decibel/100 feet. Assuming $Z = 250$ ohms as a first approximation, the formula for L_c gives $d_i = 0.068$ inch. Use No. 14 AWG wire ($d_i = 0.064$ and $\lambda/d_i = 204$). Now going to Fig. 37 and assuming that $100\delta v/c = 6$ percent is adequate, we find that $d_o/d_i = 3$ and $Z = 270$ ohms.

Recomputing, $L_c = 0.79$ decibel/100 feet. By Fig. 38, $L_p = 0.017$ at 100 megahertz for brown polyethylene. Using the same material at 900 megahertz, the loss is $L_i = 0.15$ decibel/100 feet.

For 200 feet, the combined conductor and dielectric loss is 1.9 decibels, to which must be added the loss of 0.5 to 1.0 decibel total for the two launchers.

Dielectric Other Than Polyethylene (Fig. 39)

Determine Z and $\delta v/c$ for polyethylene ($\epsilon_r = 2.3$) from Fig. 37. Then use Fig. 39 to find the value of d_o/d_i required for the same performance with actual dielectric constant ϵ_r. Make computation of new dielectric loss, using Fig. 38 and equation for L_i.

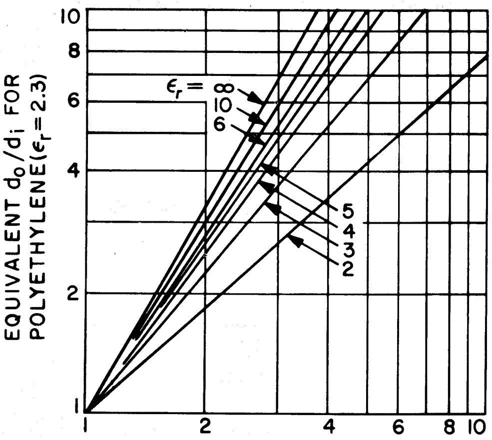

Fig. 39—Conversion chart for dielectric other than polyethylene. *Courtesy of Electronics.*

ARMY–NAVY LIST OF PREFERRED RADIO-FREQUENCY CABLES*

Class of Cables		JAN Type RG–	Inner Conductor†	Dielectric Material (Note 1)	Nominal Diameter of Dielectric (in.)	Shielding Braid	Protective Covering (Note 2)	Nominal Overall Diameter (in.)	Weight (lb/ft)	Nominal Capacitance (pF/ft)	Maximum Operating Voltage (rms)	Remarks
50 ohms	Single braid	58C/U	19/0.0071″ tinned copper	A	0.116	Tinned copper	IIa	0.195	0.029	28.5	1 900	Small-size flexible cable
		213/U	7/0.0296″ copper	A	0.285	Copper	IIa	0.405	0.120	29.5	5 000	Medium-size flexible cable (formerly RG–8A/U)
		215/U	7/0.0296″ copper	A	0.285	Copper	IIa, with armor	0.475 max.	0.160	29.5	5 000	Same as RG–213/U, but with armor (formerly RG–10A/U)
		218/U	0.195″ copper	A	0.680	Copper	IIa	0.870	0.491	29.5	11 000	Large-size low-attenuation high-power transmission line (formerly RG–17A/U)
		219/U	0.195″ copper	A	0.680	Copper	IIa, with armor	0.945 max.	0.603	29.5	11 000	Same as RG–218/U, but with armor (formerly RG–18A/U)
		220/U	0.260″ copper	A	0.910	Copper	IIa	1.120	0.745	29.5	14 000	Very-large low-attenuation high-power transmission cable (formerly RG–19A/U)
		221/U	0.260″ copper	A	0.910	Copper	IIa, with armor	1.195 max.	0.925	29.5	14 000	Same as RG–220/U, but with armor (formerly RG–20A/U)
	Double braid	55B/U	0.032″ silvered copper	A	0.116	Silvered copper	IIIa	0.206	0.032	28.5	1 900	Small-size flexible cable
		212/U	0.0556″ silvered copper	A	0.185	Silvered copper	IIa	0.332	0.093	28.5	3 000	Small-size microwave cable (formerly RG–5B/U)
		214/U	7/0.0296″ silvered copper	A	0.285	Silvered copper	IIa	0.425	0.158	30.0	5 000	Special medium-size flexible cable (formerly RG–9B/U)
		217/U	0.106″ copper	A	0.370	Copper	IIa	0.545	0.236	29.5	7 000	Medium-size power transmission line (formerly RG–14A/U)
		223/U	0.035″ silvered copper	A	0.116	Silvered copper	IIa	0.216	0.036	28.5	1 900	Small-size flexible cable (formerly RG–55A/U)
		224/U	0.106″ copper	A	0.370	Copper	IIa, with armor	0.615 max.	0.282	29.5	7 000	Same as RG–217/U, but with armor (formerly RG–74A/U)
75 ohms	Single braid	11A/U	7/26 AWG tinned copper	A	0.285	Copper	IIa	0.412	—	20.5	5 000	Medium-size flexible video cable
		12A/U	7/26 AWG tinned copper	A	0.285	Copper	IIa, with armor	0.475	—	20.5	5 000	Similar to RG–11A/U, but with armor
		34B/U	7/0.0249″ copper	A	0.460	Copper	IIa	0.630	0.231	21.5	6 500	Large-size high-power low-attenuation flexible cable
		35B/U	0.1045″ copper	A	0.680	Copper	IIa, with armor	0.945 max.	0.480	21.5	10 000	Large-size high-power low-attenuation video and communication cable

Class of Cables		JAN Type RG–	Inner Conductor†	Dielectric Material (Note 1)	Nominal Diameter of Dielectric (in.)	Shielding Braid	Protective Covering (Note 2)	Nominal Overall Diameter (in.)	Weight (lb/ft)	Nominal Capacitance (pF/ft)	Maximum Operating Voltage (rms)	Remarks
75 ohms *Continued*	Single braid	59B/U	0.0230″ copper-covered steel	A	0.146	Copper	IIa	0.242	—	21.0	2 300	General-purpose small-size video cable
		84A/U	0.1045″ copper	A	0.680	Copper	IIa, with lead sheath	1.000	1.325	21.5	10 000	Same as RG–35B/U, except lead sheath instead of armor for underground installations
		85A/U	0.1045″ copper	A	0.680	Copper	IIa, with lead sheath and special armor	1.565 max.	2.910	21.5	10 000	Same as RG–84A/U, with special armor for underground installations
		164/U	0.1045″ copper	A	0.680	Copper	IIa	0.870	0.490	21.5	10 000	Same as RG–35B/U, except without armor
		307A/U	17/0.0058″ silvered copper	A Foamed	0.029	Silvered copper	IIIa	0.270	—	20	400	
	Double braid	6A/U	21 AWG copper-covered steel	A	0.185	Inner: silvered copper. Outer: copper	IIa	0.332	—	20.0	2 700	Small-size video and communication cable
		216/U	7/0.0159″ tinned copper	A	0.285	Copper	IIa	0.425	0.121	20.5	5 000	Medium-size flexible video and communication cable (formerly RG–13A/U)
High temperature	Single braid	144/U	7/0.0179″ silvered copper-covered steel	F1	0.285	Silvered copper	Teflon-tape moisture seal with double-braid type-V jacket	0.410	0.120	20.5	5 000	Similar to RG–11A/U, except cable core is teflon. $Z=75$ ohms
		178B/U	7/0.004″ silvered copper-covered steel	F1	0.034	Silvered copper	IX	0.075 max.	—	29.0	1 000	$Z=50$ ohms
		179B/U	Same as above	F1	0.063	Silvered copper	IX	0.105	—	20.0	1 200	
		180B/U	Same as above	F1	0.102	Silvered copper	IX	0.145	—	15.5	1 500	$Z=95$ ohms
		187A/U	7/0.004″ annealed silvered copper-covered steel	F1	0.060	Silvered copper	VII	0.110	—	—	1 200	Miniaturized cable. $Z=75$ ohms
		195A/U	Same as RG–187A/U	F1	0.102	Silvered copper	VII	0.155	—	—	1 500	Miniaturized cable. $Z=95$ ohms
		196A/U	Same as RG–187A/U	F1	0.034	Silvered copper	VII	0.080	—	—	1 000	Miniaturized cable. $Z=50$ ohms
		211A/U	0.190″ copper	F1	0.620	Copper	Same as RG–144/U	0.730	0.450	29.0	7 000	Semiflexible cable operating at −55°C to +200°C (formerly RG–117A/U). $Z=50$ ohms

Class of Cables		JAN Type RG–	Inner Conductor†	Dielectric Material (Note 1)	Nominal Diameter of Dielectric (in.)	Shielding Braid	Protective Covering (Note 2)	Nominal Overall Diameter (in.)	Weight (lb/ft)	Nominal Capacitance (pF/ft)	Maximum Operating Voltage (rms)	Remarks
High temperature *Continued*	Single braid	228A/U	0.190″ copper	F1	0.620	Copper	Teflon-tape moisture seal with double-braid type-V jacket, with armor	0.795	0.600	29.0	7 000	Same as RG–211A/U, but with armor (formerly RG–118A/U). $Z=50$ ohms
		302/U	0.025″ silvered copper-covered steel	F1	0.146	Silvered copper	IX	0.206	—	21.0	2 300	$Z=75$ ohms
		303/U	0.039″ silvered copper-covered steel	F1	0.116	Silvered copper	IX	0.170	—	28.5	1 900	$Z=50$ ohms
		304/U	0.059″ silvered copper-covered steel	F1	0.185	Silvered copper	IX	0.280	—	28.5	3 000	$Z=50$ ohms
		316/U	7/0.0067″ annealed silvered copper-covered steel	F1	0.060	Silvered copper	IX	0.102	—	—	1 200	Miniaturized cable. $Z=50$ ohms
	Double braid	115/U	7/0.028″ silvered copper	F2	0.250	Silvered copper	Same as RG–144/U	0.375	—	29.5	5 000	Medium-size cable for use where expansion and contraction are a major problem. $Z=50$ ohms
		142B/U	0.039″ silvered copper-covered steel	F1	0.116	Silvered copper	IX	0.195	—	28.5	1 900	Small-size flexible cable. $Z=50$ ohms
		225/U	7/0.0312″ silvered copper	F1	0.285	Silvered copper	Same as RG–144/U	0.430	0.176	29.5	5 000	Semiflexible cable operating at −55°C to +200°C (formerly RG–87A/U). $Z=50$ ohms
		226/U	19/0.0254″ silvered copper wire	F2	0.370	Copper	Same as RG–144/U	0.500	0.247	29.0	7 000	Medium-size cable for use where expansion and contraction are a major problem (formerly RG–94A/U). $Z=50$ ohms
		227/U	7/0.0312″ silvered copper	F1	0.285	Silvered copper	Same as RG–228A/U	0.490	0.224	29.5	5 000	Same as RG–225/U, but with armor (formerly RG–116/U). $Z=50$ ohms
Pulse	Single braid	26A/U	19/0.0117″ tinned copper	E	0.288	Tinned copper	IV, with armor	0.505	0.189	50.0	10 000	High-voltage cable. $Z=48$ ohms
		27A/U	19/0.0185″ tinned copper	D	0.455	Tinned copper	IV, with armor	0.670	0.304	50.0	15 000 peak	Large-size cable. $Z=48$ ohms
	Double braid	25A/U	19/0.0117″ tinned copper	E	0.288	Tinned copper	IV	0.505	0.205	50.0	10 000	High-voltage cable. $Z=48$ ohms
		28B/U	19/0.0185″ tinned copper	D	0.455	Inner: tinned copper. Outer: galvanized steel	IV	0.750	0.370	50.0	15 000 peak	Large-size cable. $Z=48$ ohms
		64A/U	19/0.0117″ tinned copper	E	0.288	Tinned copper	IV	0.475 max.	0.205	50.0	10 000	Medium-size cable. $Z=48$ ohms

Class of Cables		JAN Type RG–	Inner Conductor†	Dielectric Material (Note 1)	Nominal Diameter of Dielectric (in.)	Shielding Braid	Protective Covering (Note 2)	Nominal Overall Diameter (in.)	Weight (lb/ft)	Nominal Capacitance (pF/ft)	Maximum Operating Voltage (rms)	Remarks
Pulse *Continued*	Double braid	156/U	7/21 AWG tinned copper	First layer A; second layer H	0.285	Inner: tinned copper. Outer: galvanized steel. Tinned copper outer shield	IIa	0.540	0.211	30.0	10 000	Taped inner layers, first layer type K and second layer type A-1R, between the outer braid of the outer conductor and the tinned copper shield. Triaxial pulse cables. $Z=50$ ohms
		157/U	19/24 AWG tinned copper	First layer H; second layer A; third layer H	0.455		IIa	0.725	0.317	38.0	15 000	
		158/U	37/21 AWG tinned copper		0.455		IIa	0.725	0.380	78.0	15 000	Same as above, except $Z=25$ ohms
		190/U	19/0.0117″ tinned copper	First layer H; second layer J; third layer H	0.380	Same as above	VIII over one wrap of type K	0.700	0.353	50.0	15 000	Taped inner layers, two wraps of type K and two wraps of type L between the outer braid and the tinned copper shield. Pulse cable. $Z=50$ ohms
		191/U	30 AWG tinned copper; single braid over supporting elements; 0.485″ max.		1.065	Same as above	Same as above	1.460	1.469	85.0	15 000	Same as RG–190/U, except $Z=25$ ohms
	Four braids	88/U	19/0.0117″ tinned copper	E	0.288	Tinned copper	IIa	0.515	—	50.0	10 000	Medium-size multishielded high-voltage cable. $Z=48$ ohms
Low capacitance	Single braid	62A/U	0.0253″ solid copper-covered steel	A	0.146	Copper	IIa	0.242	0.382	14.5	750	$Z=93$ ohms
		63B/U	0.0253″ copper-covered steel	A	0.285	Copper	IIa	0.405	0.082	10.0	1 000	Medium-size low-capacitance air-spaced cable. $Z=125$ ohms
		79B/U	0.0253″ copper-covered steel	A	0.285	Copper	IIa, with armor	0.475 max.	0.138	10.0	1 000	Same as RG–63B/U, but with armor. $Z=125$ ohms
	Double braid	71B/U	0.0253″ copper-covered steel	A	0.146	Tinned copper	IIIa	0.250 max.	—	14.5	750	Low-capacitance cable. $Z=93$ ohms
High attenuation	Single braid	301/U	7/0.0203″ Karma wire	F1	0.185	Karma wire	IX	0.245	—	29.0	3 000	High-attenuation cable. $Z=50$ ohms
High delay	Single braid	65A/U	No. 32 Formex F. Helix diameter 0.128″	A	0.285	Copper	IIa	0.405	0.096	44.0	1 000	High-impedance video cable; high-delay line. $Z=950$ ohms. (Refer to Note 3.)
Twin conductor	Single braid	57A/U	Each conductor 7/0.0285″ plain copper	A	0.472	Tinned copper	IIa	0.625	0.225	17.0	3 000	$Z=95$ ohms
		130/U		A	0.472	Tinned copper	I	0.625	0.220	17.0	8 000	Same as RG–57A/U, except inner conductors are twisted to improve flexibility. $Z=95$ ohms
		131/U		A	0.472	Tinned copper	I, with aluminum armor	0.710	0.295	17.0	8 000	Same as RG–130/U, but with armor. $Z=95$ ohms

Class of Cables	JAN Type RG-	Inner Conductor†	Dielectric Material (Note 1)	Nominal Diameter of Dielectric (in.)	Shielding Braid
Double braid	22B/U	Each conductor 7/0.0152″ copper	A	0.285	Tinned copper
	111A/U		A	0.285	Tinned copper
Twin coaxial	181/U	Each conductor 7/26 AWG copper	A	0.210	Copper inner braids and common braid

* From "RF Transmission Lines and Fittings," MIL–HDBK–216, 4 January 1962, revised 18 May 1965. Requirements for listed cables are in Specification MIL–C–17.

† Diameter of strands given in inches. As, 7/0.0296″ = 7 strands, each 0.0296-inch diameter.

Note 1—Dielectric materials: A = Polyethylene, *D* = Layer of synthetic rubber between two layers of conducting rubber, E = Layer of conducting rubber plus two layers of synthetic rubber, F1 = Solid polytetrafluoroethylene (Teflon), F2 = Semisolid or taped polytetrafluoroethylene (Teflon), H = Conducting synthetic rubber, and J = Insulating butyl rubber.

ATTENUATION AND POWER RATING OF LINES AND CABLES

Attenuation

Figure 40 illustrates the attenuation of general-purpose radio-frequency lines and cables up to their practical upper frequency limit. Most of these are coaxial-type lines, but waveguide and microstrip are included for comparison.

The following notes are applicable to this figure.

(**A**) For the RG-type cables, only the number is given (for instance, the curve for RG-218/U is labeled 218. Refer to table on pages 24-32–24-41.) The data on RG-type cables are taken mostly from, "RF Transmission Lines and Fittings," MIL–HDBK–216, 4 January 1962, revised 18 May 1965, and from "Solid Dielectric Transmission Lines," Electronic Industries Association Standard RS–199, December 1957.

Some approximation is involved in order to simplify the figure. Thus, where a single curve is labeled with several type numbers, the actual attenuation of each individual type may be slightly different from that shown by the curve.

(**B**) The curves for rigid copper coaxial lines are labeled with the diameter of the line only, as $\frac{7}{8}''C$. These have been computed for the lines listed in "Rigid Coaxial Transmission Lines, 50 Ohms," Electronic Industries Association Standard RS–225, August 1959. The computations considered the copper losses only, on the basis of a resistivity $\rho = 1.724$ microhm-centimeters; a derating of 20 percent has been applied to allow for imperfect surface, presence of fittings, etc., in long installed lengths. Relative attenuations of the different sizes are

$$A_{6\frac{1}{8}''} \approx 0.13 A_{\frac{7}{8}''}$$

$$A_{3\frac{1}{8}''} \approx 0.26 A_{\frac{7}{8}''}$$

$$A_{1\frac{5}{8}''} \approx 0.51 A_{\frac{7}{8}''}.$$

(**C**) Typical curves are shown for three sizes of 50-ohm semirigid cables such as Styroflex, Spiroline, Heliax, Alumispline, etc. These are labeled by size in inches, as $\frac{7}{8}''S$.

(**D**) The microstrip curve is for Teflon-impregnated Fibreglas dielectric $\frac{1}{16}$-inch thick and conductor strip $\frac{7}{32}$-inch wide.

(**E**) Shown for comparison is the attenuation in the $TE_{1,0}$ mode of 5 sizes of brass waveguide. The resistivity of brass was taken as $\rho = 6.9$ microhm-centimeters, and no derating was applied. For copper or silver, attenuation is about half that for brass. For aluminum, attenuation is about two-thirds that for brass.

Protective Covering (Note 2)	Nominal Overall Diameter (in.)	Weight (lb/ft)	Nominal Capacitance (pF/ft)	Maximum Operating Voltage (rms)	Remarks
IIa	0.420	0.116	16.0	1 000	Small-size balanced twin-conductor cable. $Z = 95$ ohms
IIa, with armor	0.490 max.	0.145	16.0	1 000	Same as RG–22B/U, but with armor. $Z = 95$ ohms
IIa	0.640	—	12	3 500	Filled-to-round, unbalanced transmission cable. Twin coaxial. $Z = 125$ ohms

Note 2—Jacket types: I = Polyvinyl chloride (colored black), IIa = Noncontaminating synthetic resin, IIIa = Noncontaminating synthetic resin (colored black), IV = Chloroprene, V = Fibreglas, silicone-impregnated varnish, VII = Polytetrafluoroethylene, VIII = Polychloroprene, and IX = Fluorinated ethylene propylene.

Note 3—For RG-65A/U, delay = 0.042 microsecond per foot at 5 megahertz; dc resistance = 7.0 ohms/foot.

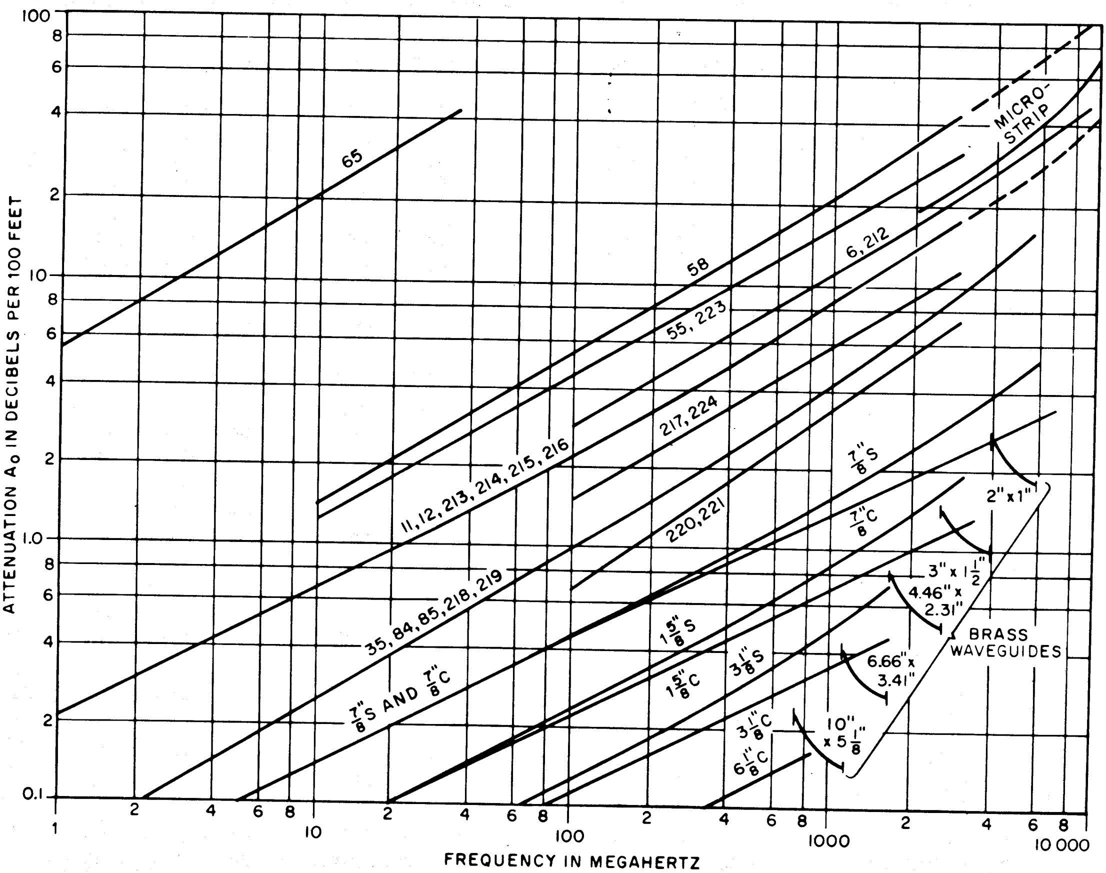

Fig. 40—Attenuation of cables.

Power Rating

Figure 41 shows the approximate power transmitting capabilities of various coaxial-type lines. The following notes are applicable.

(**A**) Identification of the curves for the RG-type cables is the same as in Fig. 40. The data for these cables are from the same sources. For polyethylene cables, an inner-conductor maximum temperature of 80 degrees Celsius is specified (**G**). For high-temperature cables (types 211, 228; 225, 227) the inner-conductor temperature is 250 degrees Celsius.

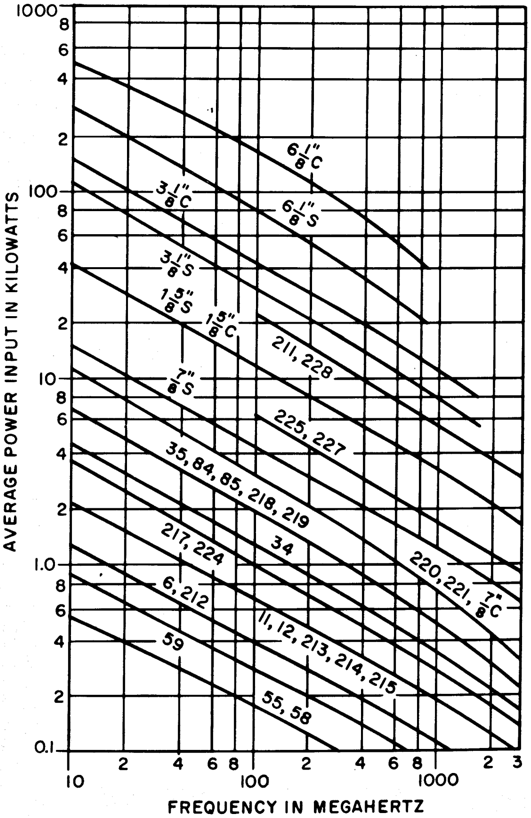

Fig. 41—Power rating of cables.

(**B**) The curves for 50-ohm rigid coaxial line are labeled with the diameter of the line only, as 7/8″*C*. These are rough estimates based largely on miscellaneous charts published in catalogs.

(**C**) For Styroflex, Spiroline, Heliax, Alumispline, etc., cables, refer to (**C**) above.

(**D**) The curves are for unity voltage standing-wave ratio. Safe operating power is inversely proportional to swr expressed as a numerical ratio greater than unity. Do not exceed maximum operating voltage (see pp. 24-25 and 24-32–24-40).

(**E**) An ambient temperature of 40 degrees Celsius is assumed.

(**F**) The 4 curves meeting the 100-watt ordinate may be extrapolated: at 3000 megahertz for 55, 58, power is 28 watts; for 59, power is 44 watts; and for 6, 212, power is 58 watts.

(**G**) Electronic Industries Association Standard RS–199 states that operation of a polyethylene dielectric cable at a center-conductor temperature in excess of 80 degrees Celsius is likely to cause permanent damage to the cable. Where practicable, and particularly where continuous flexing is required, it is recommended that a cable be selected which, in regular operation, will produce a center-conductor temperature not greater than 65 degrees Celsius. Rating factors for various operating temperatures are given in the following table. Multiply points on the power-rating curve by the factors in the table to determine power rating at operating conditions.

Derating Factor

Ambient Temperature (°C)	Maximum Allowable Center-Conductor Temperature (°C)			
	80	75	70	65
40	1.0	0.86	0.72	0.59
50	0.72	0.59	0.46	0.33
60	0.46	0.33	0.22	0.10
70	0.20	0.09	0	—
80	0	—	—	—